Adaptive Equalization With Time Lenses

Wolfgang Freude

Institute of High-Frequency and Quantum Electronics (IHQ), University of Karlsruhe, Germany

Universität Karlsruhe (TH)
Institut für Hochfrequenztechnik und Quantenelektronik (IHQ)
http://www.ihq.uni-karlsruhe.de

Institute Seminar December 13, 2006

Outline

- Spatial Fourier transform
  - Farfield transform
  - Lens transform
  - Gaussian beam
- Dispersion in linear waveguide
  - Chirp in dispersive waveguide
  - Analogy to Gaussian beam
- Adaptive dispersion compensation
  - Time lens transform
  - Imaging with 2nd-order dispersion
- Experiment

Outline

- Spatial Fourier transform
  - Farfield transform
  - Lens transform
  - Gaussian beam
- Dispersion in linear waveguide
  - Chirp in dispersive waveguide
  - Analogy to Gaussian beam
- Adaptive dispersion compensation
  - Time lens transform
  - Imaging with 2nd-order dispersion
- Experiment

Fourier Transform by Propagation — Solution of Wave Equation

Homogeneous medium, monochromatic fields, real k_x ($k_y = 0$), superimposing all homogeneous and evanescent plane waves, Dirichlet boundary condition $\Psi(x, z = 0)$ in plane $z = 0$:

$$\widetilde{\Psi}_0(\xi) = \int_{-\infty}^{+\infty} \Psi(x,0)\,\mathrm{e}^{+\mathrm{j}\,2\pi\xi x}\,\mathrm{d}x\,, \quad \Psi(x,0) = \int_{-\infty}^{+\infty} \widetilde{\Psi}_0(\xi)\,\mathrm{e}^{-\mathrm{j}\,2\pi\xi x}\,\mathrm{d}\xi$$

Spatial Fourier transform $\widetilde{\Psi}_0(\xi)$, spatial frequency ξ in direction of x-axis. $\widetilde{\Psi}_0(\xi)$ defined by BC $\Psi(x,0) \to$ components $\vec{E}$, $\vec{H}$.

Farfield "performs" a Fourier transform on a circle d (here: $n = 1$):

$$\Psi(x,z) = \mathrm{e}^{\mathrm{j}\,\frac{\pi}{4}}\,\mathrm{e}^{-\mathrm{j}\,k_0 d}\,\frac{\cos\gamma}{\sqrt{\lambda d}}\,\widetilde{\Psi}_0\Big(\frac{x}{\lambda d}\Big), \qquad \cos\gamma = z/d$$

$$d = \sqrt{x^2 + z^2}\,, \qquad d\lambda \gg \pi x_M^2\,, \qquad d/\lambda \gg 0.14/\cos^2\gamma$$

Field in planes $z = \text{const}$ required $\to$ weird phase factors in $\widetilde{\Psi}_0(\cdots)$, Fresnel transform needed (implies a paraxial approximation).

Freude, W.: 'Analyse von Lichtwellenleitern aus dem Nah- und Fernfeld (Analysis of light waveguides from nearfield and farfield data)'. Habilitationsschrift, Karlsruhe, 04.06.1986. Eq. (F3-14) — The full text is available: **http://www2.ihq.uni-karlsruhe.de/main/staff/freude/habil86-freude.pdf**

Outline

- Spatial Fourier transform
  - Farfield transform
  - Lens transform
  - Gaussian beam
- Dispersion in linear waveguide
  - Chirp in dispersive waveguide
  - Analogy to Gaussian beam
- Adaptive dispersion compensation
  - Time lens transform
  - Imaging with 2nd-order dispersion
- Experiment

Fourier Transform by Propagation — Thin Lens $\exp\left(\mathrm{j}\,\frac{x^2}{\lambda F}\right)$

Lens "performs" spatial Fourier transform (focal length F):

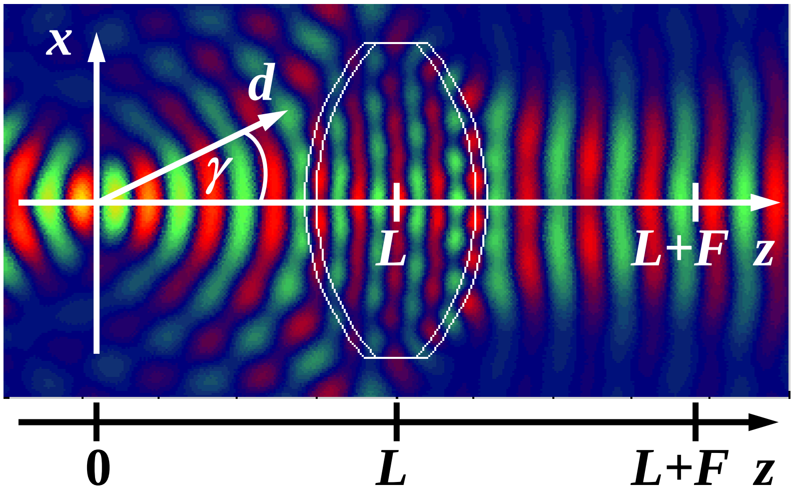
$$\Psi(x, L+F) = \mathrm{e}^{\mathrm{j}\frac{\pi}{4}} \frac{\mathrm{e}^{-\mathrm{j}\,k_0(L+F)}}{\sqrt{\lambda L}} \mathrm{e}^{-\mathrm{j}\,\pi\frac{x^2(1-L/F)}{\lambda F}} \,\widetilde{\Psi}_0\left(\tfrac{x}{\lambda L}\right)$$

Spatial Fourier transform:

$$\widetilde{\Psi}_0(\xi) = \int_{-\infty}^{+\infty} \Psi(x,0)\,\mathrm{e}^{+\mathrm{j}\,2\pi\xi x}\,\mathrm{d}x$$

In back focal plane ($z = L + F$) a photo-sensitive film records the spatial power spectrum:

$$\left|\Psi(x, L+F)\right|^2 = \frac{1}{\lambda L}\left|\widetilde{\Psi}_0\left(\tfrac{x}{\lambda L}\right)\right|^2$$



only scaling depends on L!

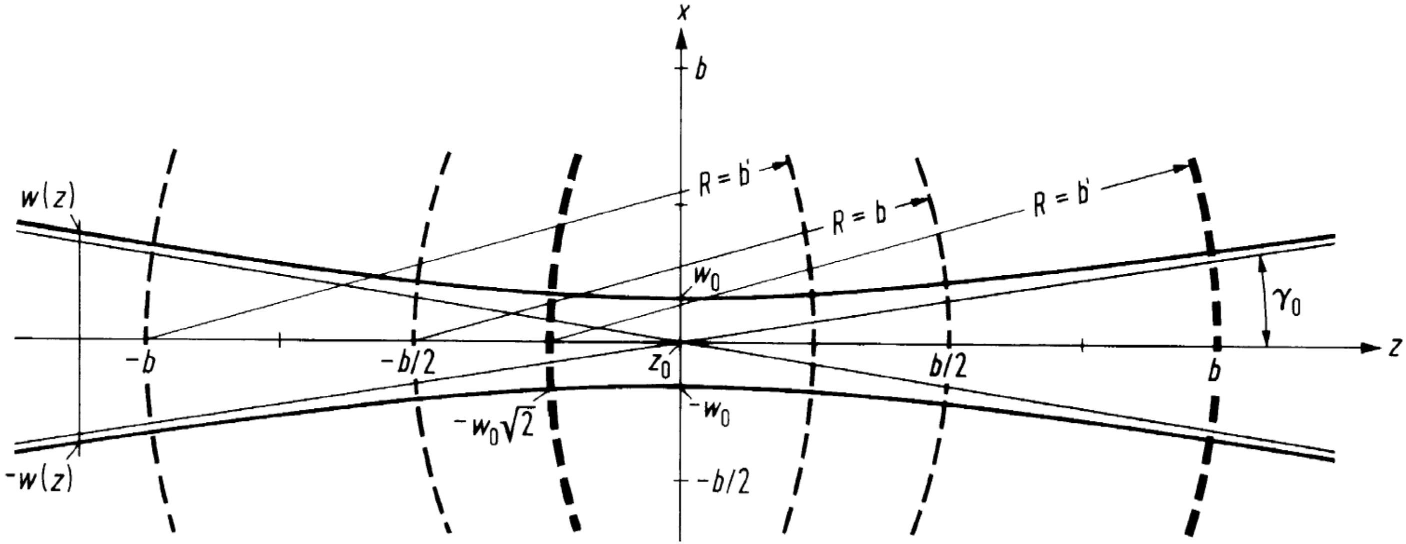
Fields in front and back focal plane of lens ($L = F$) related by a spatial Fourier transform:

$$\Psi(x, 2F) = \mathrm{e}^{\mathrm{j}\frac{\pi}{4}} \frac{\mathrm{e}^{-\mathrm{j}\,2k_0 F}}{\sqrt{\lambda F}} \,\widetilde{\Psi}_0\left(\tfrac{x}{\lambda F}\right)$$

Outline

- Spatial Fourier transform
  - Farfield transform
  - Lens transform
  - Gaussian beam
- Dispersion in linear waveguide
  - Chirp in dispersive waveguide
  - Analogy to Gaussian beam
- Adaptive dispersion compensation
  - Time lens transform
  - Imaging with 2nd-order dispersion
- Experiment

Gaussian Beam — Amplitude and Phase Surfaces



2D scalar wave equation, ansatz $\Psi(t, \vec{r}) = \Psi(x, z)\,\exp[\mathrm{j}(\omega t - kz)]$:

0, paraxial

$$\left(\frac{\partial^2}{\partial x^2} + \cancel{\frac{\partial^2}{\partial z^2}} - \mathrm{j}\,2k\,\frac{\partial}{\partial z}\right)\Psi(x, z) = 0\,, \quad \text{paraxial approximation } \frac{\partial^2}{\partial z^2} \ll 2k\,\frac{\partial}{\partial z}$$

Lowest-order solution of paraxial wave equation is Gaussian beam:

$$\Psi(x, z) = \Psi_N(z)\,\mathrm{e}^{-\frac{x^2}{w^2(z, z_0)}}\,\mathrm{e}^{-\mathrm{j}\,\frac{1}{2}k\frac{x^2}{R(z, z_0)}}\,\mathrm{e}^{-\mathrm{j}\,k(z - z_0)}\,\mathrm{e}^{\mathrm{j}\,\frac{1}{2}\arctan\left(\frac{2(z - z_0)}{b}\right)},$$

$$\Psi_N^2(z) = \sqrt{2}\Big/\left(w_0\sqrt{\pi}\right)\cdot\left[w_0/w(z, z_0)\right]$$

Outline

- Spatial Fourier transform
  - Farfield transform
  - Lens transform
  - Gaussian beam
- Dispersion in linear waveguide
  - Chirp in dispersive waveguide
  - Analogy to Gaussian beam
- Adaptive dispersion compensation
  - Time lens transform
  - Imaging with 2nd-order dispersion
- Experiment

Dispersion in a Linear Waveguide with Transfer Function $\breve{h}_z(f)$

Complex field envelope $A(t,z)$, frequency offset $\Delta f = f - f_0$:

$$\underline{a}(t,z) = A(t,z)\,\mathrm{e}^{\mathrm{j}(\omega_0 t - \beta^{(0)} z)} = \int_{-\infty}^{+\infty} \breve{\underline{a}}(f)\,\breve{h}_z(f)\,\mathrm{e}^{\mathrm{j}\,2\pi f t}\,\mathrm{d}f,\ \breve{h}_z(f) = \mathrm{e}^{-\mathrm{j}\,\beta(\omega) z},$$

$$A(t,z) = \int_{-\infty}^{+\infty} \breve{\underline{a}}(f)\,\mathrm{e}^{-\mathrm{j}\left[\Delta\omega\,\beta^{(1)} + \frac{(\Delta\omega)^2}{2!}\beta^{(2)} + \frac{(\Delta\omega)^3}{3!}\beta^{(3)} + \ldots\right] z}\,\mathrm{e}^{\mathrm{j}\,\Delta\omega t}\,\mathrm{d}(\Delta f)$$

Derivatives: $\left(-\mathrm{j}\,\frac{\partial}{\partial t}\right)^n A(t,z) \circ\!\!-\!\!\bullet\ (\Delta\omega)^n \breve{A}(f,z)$,

$$\frac{\partial}{\partial z} A(t,z) \circ\!\!-\!\!\bullet\ -\mathrm{j}\left[\sum\nolimits_{n\geq 1} \frac{(\Delta\omega)^n}{n!}\,\beta^{(n)}\right] \breve{A}(f,z)$$

$(\Delta f) \rightarrow (\partial/\partial t)$, retard. time $t' = t - \beta^{(1)} z$, $A'(t',z) = A(t' + \beta^{(1)} z, z)$:

$$\frac{\partial}{\partial z} A'(t',z) = -\mathrm{j}\left[\sum\nolimits_{n\geq 2} \frac{1}{n!}\,\beta^{(n)} \left(-\mathrm{j}\,\frac{\partial}{\partial t'}\right)^n\right] A'(t',z)$$

Bahaa, E. A.; Mansour, I.: Transmission of pulse sequences through monomode fibers. Appl. Optics 21 (1982), 4219–4222

Freude, W.: Meßverfahren der optischen Nachrichtentechnik: Lichtwellenleiter. Vorlesungsmanuskript 1983–1989. Anhang F, Seite F46
— The full text is available: http://www2.ihq.uni-karlsruhe.de/main/staff/freude/mont_1983-89.pdf

M. Nakazawa, T. Hirooka, F. Futami, S. Watanabe: Ideal distortion-free transmission using optical Fourier transformation and Fourier transform-limited optical pulses. IEEE Photon. Technol. Lett. 16 (2004) 1059–1061

T. Hirooka, M. Nakazawa: Optical adaptive equalization of high-speed signals using time-domain optical Fourier transformation. J. Lightw. Technol. 24 (2006) 2530–2540

Dispersion in a Linear Lossless Waveguide

Field propagation equation in lossless but dispersive waveguide:

$$\frac{\partial}{\partial z} A(t,z) = -\mathrm{j}\left[\sum_{n\geq 1} \frac{1}{n!}\beta^{(n)}\left(-\mathrm{j}\frac{\partial}{\partial t}\right)^n\right] A(t,z)$$

Fourier transform solution:

$$\frac{\partial}{\partial z} \bar{A}(f,z) = -\mathrm{j}\left[\sum_{n\geq 1} \frac{(\Delta\omega)^n}{n!}\beta^{(n)}\right] \bar{A}(f,z),$$

$$\bar{A}(f,z) = \bar{A}(f,0)\exp\left\{-\mathrm{j}\left[\sum_{n\geq 1} \frac{(\Delta\omega)^n}{n!}\beta^{(n)}\right] z\right\}$$

Power spectrum $|\bar{A}(f,z)|^2$ remains unchanged. Why? Widening of impulse (narrows the power spectrum) counteracted and exactly balanced by chirp (widens the power spectrum).

It is the chirp which considerably distorts the impulse shape in the time domain.

Dispersion compensation means getting rid of this chirp.

Outline

- Spatial Fourier transform
  - Farfield transform
  - Lens transform
  - Gaussian beam
- Dispersion in linear waveguide
  - Chirp in dispersive waveguide
  - Analogy to Gaussian beam
- Adaptive dispersion compensation
  - Time lens transform
  - Imaging with 2nd-order dispersion
- Experiment

Gaussian Impulse — Evolution in Time and Space

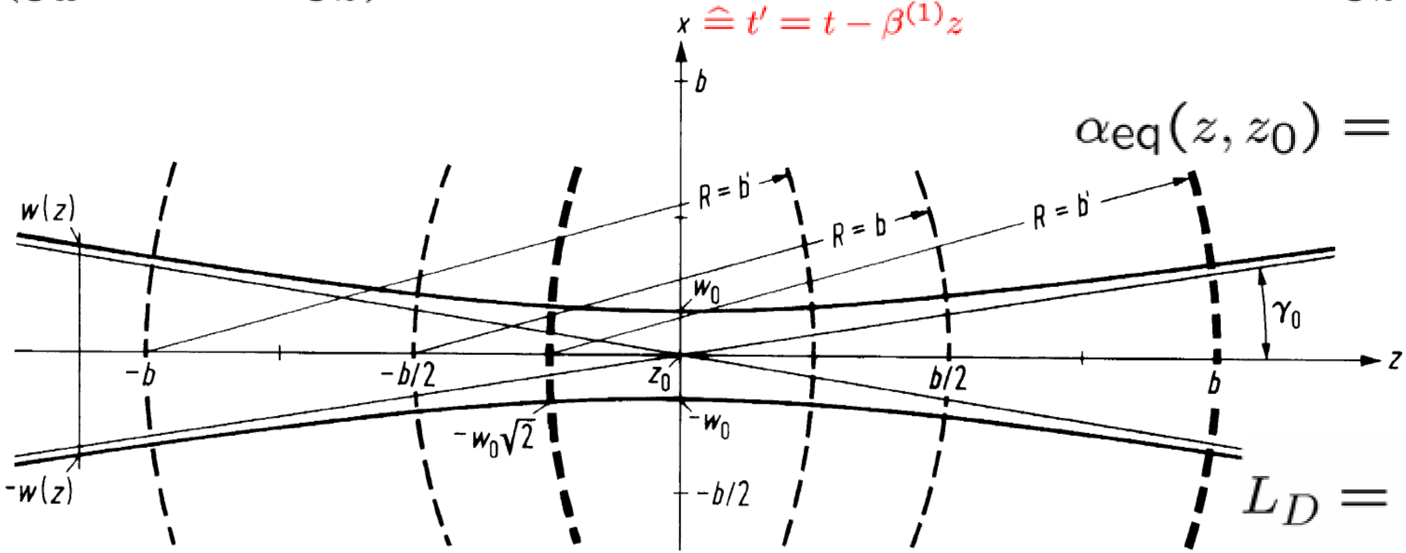
Field propagation equation in dispersive WG with $\beta^{(n\geq3)}=0$:

$$\frac{\partial}{\partial z}A'(t',z)=-\mathrm{j}\left[\sum_{n\geq2}\frac{1}{n!}\beta^{(n)}\left(-\mathrm{j}\frac{\partial}{\partial t'}\right)^{n}\right]A'(t',z)=\mathrm{j}\frac{1}{2!}\beta^{(2)}\frac{\partial^2}{\partial t'^2}A'(t',z),$$

$$\left(\frac{\partial^2}{\partial t'^2}+\mathrm{j}\frac{2}{\beta^{(2)}}\frac{\partial}{\partial z}\right)A'(t',z)=0 \qquad \text{for} \quad D,\beta^{(n\geq3)}=0\,, \quad C,\beta^{(2)}\neq0$$

2D scalar wave equation, ansatz $\Psi(t,\vec{r})=\Psi(x,z)\exp[\mathrm{j}(\omega t-k_0 z)]$:

$$\left(\frac{\partial^2}{\partial x^2}-\mathrm{j}\,2k_0\frac{\partial}{\partial z}\right)\Psi(x,z)=0\,, \quad \text{paraxial approximation} \quad \frac{\partial^2}{\partial z^2}\ll 2k_0\frac{\partial}{\partial z}$$



$$\alpha_{\text{eq}}(z,z_0)=-\frac{2\sigma_p^2}{\beta^{(2)}R(z,z_0)}$$

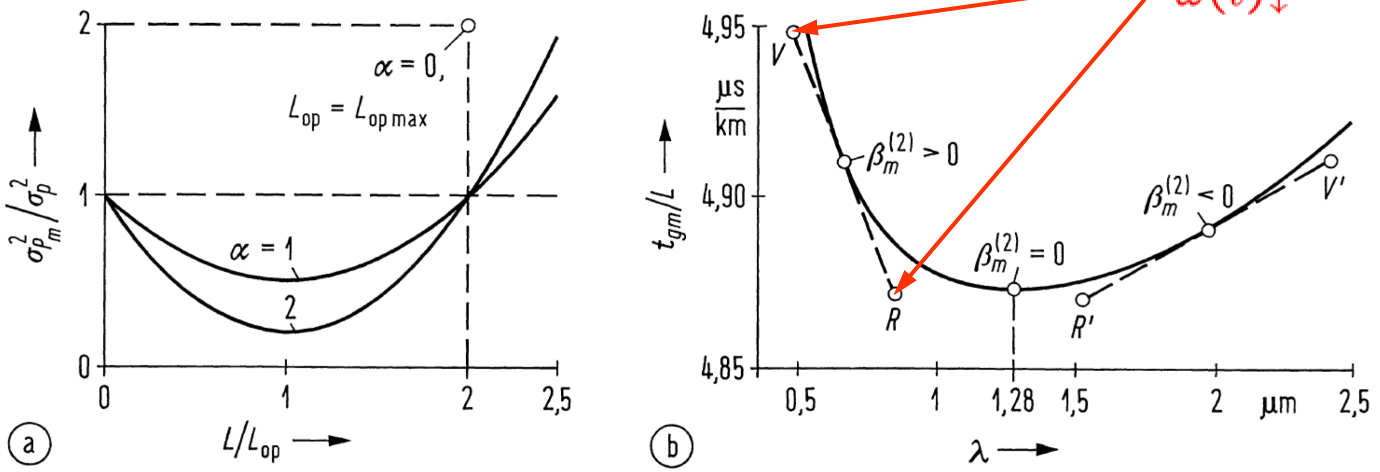
$$L_D=\frac{b}{2}=\frac{\tau_0^2}{2\left|\beta^{(2)}\right|}$$

Gaussian Impulse — Chirp, Dispersion, and Group Delay

Intuitive understanding of impulse propagation in dispersive waveguide, no source noise, initial $\alpha > 0$:

$$A(t,0) = A_s(t) = A_0\,\mathrm{e}^{-t^2/(4\sigma_p^2)}\,\underbrace{\mathrm{e}^{-\mathrm{j}\,\alpha\, t^2/(4\sigma_p^2)}}_{\downarrow} \quad \text{at } z=0$$

$$\exp\left[\mathrm{j}\,\underbrace{(-\alpha\, t)}_{\omega(t)\downarrow}\, t\right]$$



Intensity modulated monochromatic light source with chirp. (a) relative variance of received impulse as a function of the repeater length L normalized to the optimum value L_{op}. Chirp factors $\alpha = 1, 2$; (○) chirp-free source ($\alpha = 0$, $L_{\mathrm{op}} = L_{\mathrm{op\,max}}$) (b) length-related group delay as a function of λ in a „waveguide“ mode (material dispersion for undoped quartz glass, Fig. 2.6)

Outline

- Spatial Fourier transform
  - Farfield transform
  - Lens transform
  - Gaussian beam
- Dispersion in linear waveguide
  - Chirp in dispersive waveguide
  - Analogy to Gaussian beam
- Adaptive dispersion compensation
  - Time lens transform
  - Imaging with 2nd-order dispersion
- Experiment

Approach to Adaptive Dispersion Compensation

If we could generate a time signal $B(t, L+F) \sim \bar{A}(t/s, L)$ (scaling factor s, unit s/Hz), and if we chose a self-reciprocal function such that $\bar{A}(f,z) \sim A(sf,z)$, i. e., $\bar{A}(t/s,z) \sim A(t,z)$ holds, then the input impulse could be ideally recovered by a square-law detector:

$$i(t) \sim |B(t, L+F)|^2 \sim |\bar{A}(t/s, L)|^2 \sim |A(t,0)|^2$$

Chirp acquired during propagation ($|\cdots|^2$ makes distortion vanish):

$$\bar{A}(f,z) = \bar{A}(f,0) \exp\left\{-\mathrm{j}\left[\sum_{n\geq 1} \frac{(\Delta\omega)^n}{n!}\beta^{(n)}\right] z\right\}$$

Usually: DCF with $\bar{h}_{\mathrm{DCF}}(f) = \exp\left\{-\mathrm{j}\left[\sum_{n\geq 2} \frac{(\Delta\omega)^n}{n!}\beta^{(n)}_{\mathrm{DCF}}\right] L_{\mathrm{DCF}}\right\}$

Gaussian is self-reciprocal (also when multiplied with a polynomial):

$$G(t,\bar{t},\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \mathrm{e}^{-(t-\bar{t})^2/(2\sigma^2)}, \qquad \bar{G}(f,\bar{t},\sigma) = \mathrm{e}^{-\sigma^2\omega^2/2}\,\mathrm{e}^{-\mathrm{j}\,\omega\bar{t}},$$

$$G(t,0,\sigma) = \frac{1}{\sqrt{s}}\bar{G}(t/s,0,\sigma), \qquad s = 2\pi\sigma^2$$

Fourier Transform by Chirp — Time Lens $\exp\left(-\mathrm{j}\,\frac{\alpha t^2}{4\sigma_p^2}\right)$

Transfer function $\breve{h}_F(f)$ of time lens waveguide (ideal "imaging"):

$$\breve{h}_F(f) = \mathrm{e}^{-\mathrm{j}(k^{(0)}+\Delta\omega\, k^{(1)}+\frac{(\Delta\omega)^2}{2}k^{(2)})F}$$

Impulse response $h_F(t)$ of time lens waveguide ($t'_F = t - k^{(1)}F$):

$$h_F(t) = \mathrm{e}^{\mathrm{j}(\omega_0 t - k^{(0)}F)} \int_{-\infty}^{+\infty} \mathrm{e}^{-\mathrm{j}(\Delta\omega\, k^{(1)}+\frac{(\Delta\omega)^2}{2}k^{(2)})F}\,\mathrm{e}^{\mathrm{j}\,\Delta\omega\, t}\,\mathrm{d}(\Delta f)$$

$$= \mathrm{e}^{\mathrm{j}(\omega_0 t - k^{(0)}F)}\,\mathrm{e}^{\mathrm{j}\frac{t_F'^2}{2k^{(2)}F}} \int_{-\infty}^{+\infty} \mathrm{e}^{-\mathrm{j}\left(\sqrt{|k^{(2)}|/2}\,\Delta\omega - \frac{t'_F}{\sqrt{2|k^{(2)}|\,F}}\right)^2 F}\,\frac{\mathrm{d}(\Delta\omega)}{2\pi}$$

Fresnel integral: $\int_{-\infty}^{+\infty}\exp(\pm\mathrm{j}\,Ku^2)\,\mathrm{d}u = \sqrt{\frac{\pi}{K}}\exp(\pm\mathrm{j}\,\frac{\pi}{4})$ for $K > 0$

Impulse response: $h_F(t) = \mathrm{e}^{-\mathrm{j}\,\mathrm{sign}(k^{(2)})\frac{\pi}{4}}\,\frac{\mathrm{e}^{\mathrm{j}(\omega_0 t - k^{(0)}F)}}{\sqrt{2\pi|k^{(2)}|F}}\,\exp\left(\mathrm{j}\,\frac{t_F'^2}{2k^{(2)}F}\right)$

Simplification for transport ($\beta^{(n\le 1)}$) and lens waveguides ($k^{(n\le 1)}$):

$$k^{(0)} = \beta^{(0)}, \qquad k^{(1)} = \beta^{(1)}$$

Outline

- Spatial Fourier transform
  - Farfield transform
  - Lens transform
  - Gaussian beam
- Dispersion in linear waveguide
  - Chirp in dispersive waveguide
  - Analogy to Gaussian beam
- Adaptive dispersion compensation
  - Time lens transform
  - Imaging with 2nd-order dispersion
- Experiment

Fourier Transform by Chirp — Time Lens $\exp\left(-\mathrm{j}\,\frac{\alpha t^2}{4\sigma_p^2}\right)$

Complex field envelope $A(t, L)$ has spectrum:

$$\breve{A}(f, L) = \breve{A}(f, 0)\exp\left\{-\mathrm{j}\left[\sum_{n\geq 1}\frac{(\Delta\omega)^n}{n!}\,\beta^{(n)}\right]L\right\}$$

Generating a chirp with a phase modulator at $z = L$:

$$B(t, L) = A(t, L)\exp\left(\mathrm{j}\,\frac{\alpha(t-\beta^{(1)}L)^2}{4\sigma_p^2}\right)$$

Impulse resp.: $$h_F(t) = \mathrm{e}^{-\mathrm{j}\,\mathrm{sign}(k^{(2)})\frac{\pi}{4}}\,\frac{\mathrm{e}^{\mathrm{j}(\omega_0 t-\beta^{(0)}F)}}{\sqrt{2\pi|k^{(2)}|F}}\,\exp\left(\mathrm{j}\,\frac{(\overbrace{t-\beta^{(1)}F}^{t'_F})^2}{2k^{(2)}F}\right)$$

Complex field envelope $B(t, L+F)$ after lens waveguide at $z = L+F$:

$$b(t, L+F) = B(t, L+F)\,\mathrm{e}^{\mathrm{j}[\omega_0 t-\beta^{(0)}(L+F)]} = \int_{-\infty}^{+\infty} b(\tau, L)\,h_F(t-\tau)\,\mathrm{d}\tau$$

$$B(t, L+F) = \mathrm{e}^{-\mathrm{j}\,\frac{\pi}{4}\mathrm{sign}(k^{(2)})}\,\frac{\mathrm{e}^{-\mathrm{j}\,\frac{(\beta^{(1)}L)^2}{2k^{(2)}F}}}{\sqrt{2\pi|k^{(2)}|F}}\,\mathrm{e}^{\mathrm{j}\,\frac{(t-\beta^{(1)}F)^2}{2k^{(2)}F}}\,\breve{A}\left(\frac{t-\beta^{(1)}F}{2\pi k^{(2)}F}, L\right)$$

Adaptive Dispersion Compensation — Time Lens $\exp\left(-\mathrm{j}\,\frac{\alpha t^2}{4\sigma_p^2}\right)$

Complex field envelope at power detector:

$$B(t, L+F) = \mathrm{e}^{-\mathrm{j}\frac{\pi}{4}\operatorname{sign}(k^{(2)})}\,\frac{\mathrm{e}^{-\mathrm{j}\frac{(\beta^{(1)}L)^2}{2k^{(2)}F}}}{\sqrt{2\pi|k^{(2)}|F}}\,\mathrm{e}^{\mathrm{j}\frac{(t-\beta^{(1)}F)^2}{2k^{(2)}F}}\,\breve{A}\left(\frac{t-\beta^{(1)}F}{2\pi k^{(2)}F}, L\right)$$

Detector current at $z = L + F$ proportional to power spectrum:

$$i(t) = |B(t, L+F)|^2 = \frac{1}{2\pi|k^{(2)}|F}\left|\breve{A}\left(\frac{t-\beta^{(1)}F}{2\pi k^{(2)}F}, L\right)\right|^2$$

Gaussian input impulse at $z = 0$:

$$A(t, 0) = \sqrt{p_0}\,\mathrm{e}^{-\frac{t^2}{4\sigma_p^2}}, \qquad \breve{A}(f, 0) = \sqrt{4\pi\sigma_p^2\,p_0}\,\mathrm{e}^{-\sigma_p^2\omega^2}$$

Detector current for $2\sigma_p^2 = |k^{(2)}|F = \alpha k^{(2)}F \rightarrow \alpha = \operatorname{sign}(k^{(2)})$:

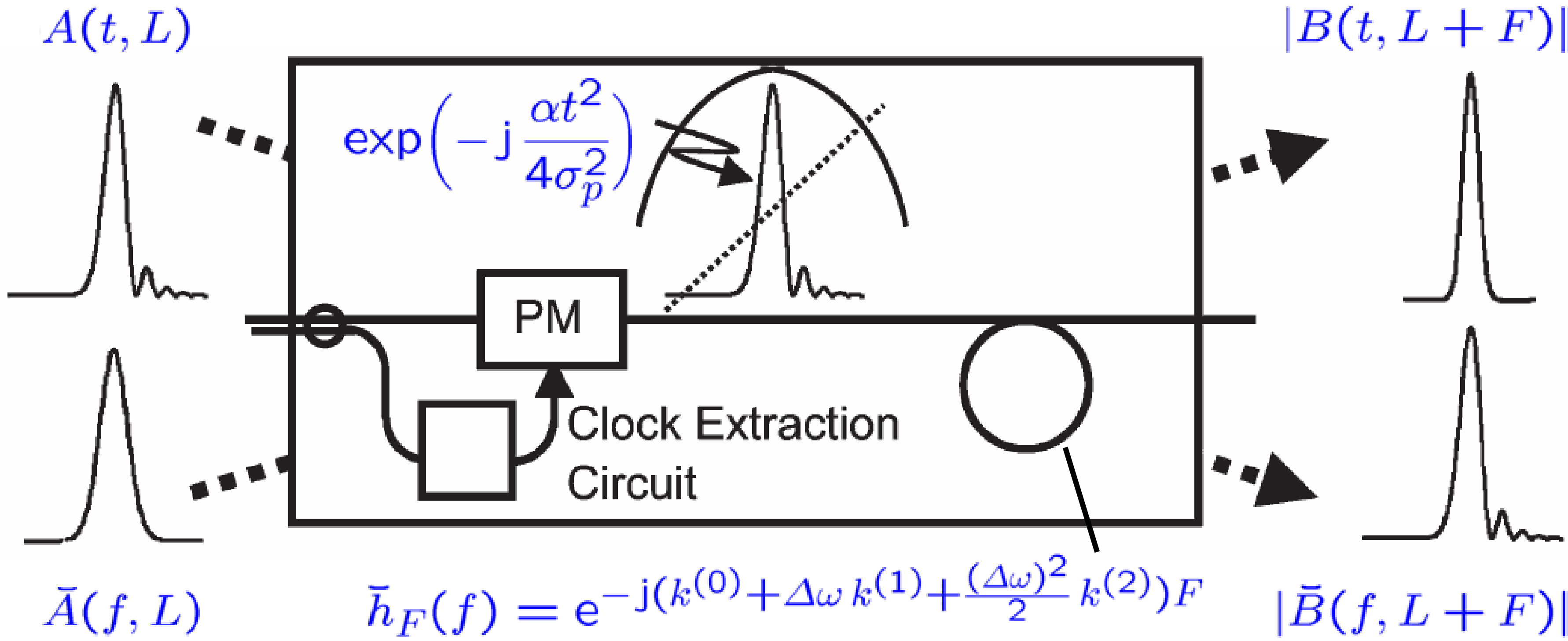
$$i(t) = \sqrt{p_0}\,\mathrm{e}^{-\frac{[t-\beta^{(1)}(L+F)]^2}{2\sigma_p^2}} \qquad \text{because} \quad |\breve{A}(f, L)|^2 = |\breve{A}(f, 0)|^2$$

This is true irrespective of $\breve{h}_L(f)$, i. e., for any order and time dependency of dispersion, and even for PMD and timing jitter!

Outline

- Spatial Fourier transform
  - Farfield transform
  - Lens transform
  - Gaussian beam
- Dispersion in linear waveguide
  - Chirp in dispersive waveguide
  - Analogy to Gaussian beam
- Adaptive dispersion compensation
  - Time lens transform
  - Imaging with 2nd-order dispersion
- Experiment

Experimental Setup 160 Gbit/s



Detector current at $z = L + F$ proportional to power spectrum:

$$i(t) = \frac{1}{2\pi|k^{(2)}|F}\left|\breve{A}\left(\frac{t-\beta^{(1)}F}{2\pi k^{(2)}F}, L\right)\right|^2 = \left|A\left(t-\beta^{(1)}(L+F), 0\right)\right|^2$$

True for Gaussian $2\sigma_p^2 = \underbrace{|k^{(2)}|F}_{\text{WG}} = \underbrace{\alpha k^{(2)} F}_{\text{PM}} \to \alpha = \text{sign}(k^{(2)})$