10. Tutorial on Optical Sources and Detectors

July 10th 2012

Problem 1: Operating regimes of an illuminated photodiode

Figure 1 shows the current-voltage characteristic of a photodiode with and without irradiation. In the following you can assume that the internal photocurrent i depends linearly on the incident optical power $i = SP_e$, where S is the responsivity of the diode.

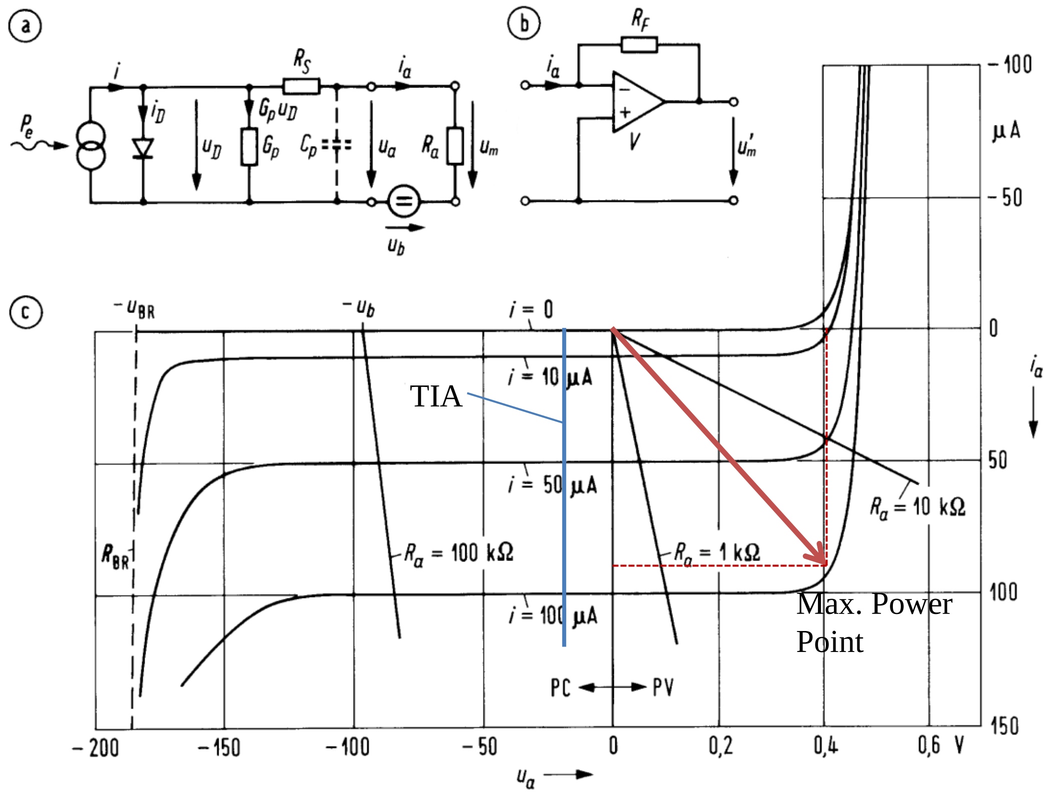

Figure 1: Equivalent circuit of a photodiode for the DC case. (a) Circuit model: P_e incident optical power, S responsivity, G_p parasisitic admittance, R_S serial resistance, R_a external resistance, u_b bias voltage, i_a and u_a current and voltage in the outer circuit. (b) Transimpedance amplifier providing zero input impedance. (c) i_a-u_a-characteristics for different photocurrents i. U_{BR} = 182V breakthrough voltage, R_{BR} differential resistance at breakthrough. Note the different scaling on the x-axis!

a) Consider first the case where the photodiode is operated with a fixed resistor R_a in the outer circuit. How should the bias voltage u_b be chosen to obtain a linear relation between the incident optical power P_e and the diode current i_a in the outer circuit.

➔ For a linear operation the photo diode should be operated in reverse bias. But well below the breakthrough limit.

b) Now the bias is fixed at u_b = 10 V, and the photocurrent i_a is determined by measuring the voltage drop $u_m = R_a\, i_a$ over the external resistor R_a. How would you choose the value of the resistor to obtain a sensitive measurement, where small changes in the incident optical power lead to large changes in the measured voltage? What are the effects on the dynamic range in this configuration, i.e., the input power range over which a linear relationship between P_e and u_m can be assumed?

➔ In general the value resistor should be chosen large, so a small current change still gives a large voltage drop according to $u_m = R_a\, i_a$. However, the larger the resistor, the smaller the dynamic range.

$u_a = R_a\, i_a - u_b$, with u_b = 10 V and i_a = 100 µA → u_a = 0 V and therefore the photodiode starts to operate in the nonlinear regime of the curve.

c) Now the resistor R_a is replaced by a transimpedance amplifier (TIA), Fig. 1 (b). Sketch the dependence of u_a over i_a for a given bias u_b into the diagram of Fig. 1 as already done for several values of R_a. Give an expression for the output voltage u_m' of the TIA in dependence of the current in the outer circuit i_a.

➔ The TIA operates in the negative feedback mode, the feedback regulates the potential between the two inputs so that the potential between the two is zero. Since one input is connected to ground, the TIA acts as a virtual ground for the circuit and the *u*-*i* dependence is a straight line.

From Kirchoffs rule we find that the output voltage of the TIA calculates to $u_m' = -R_F \cdot i_a$

d) Indicate roughly the operating point in which maximum energy can be extracted from the device in photo-voltaic operation.

➔ Pout = $u_a \cdot i_a$= max, see picture

Problem 2: Operation principles of a pin diode

Assume a one-dimensional model of the pin diode as shown in Figure 2, where the light is incident from the left. In this model all physical quantities depend only on the *x*-coordinate. The cross-sectional area of the photodiode is denoted as *A*. Assume a complete ionization of donor/acceptor impurities ("Störstellenerschöpfung"). In the following the device is strongly biased in reverse direction such that all carriers in the intrinsic zone travel at their saturation drift velocity.

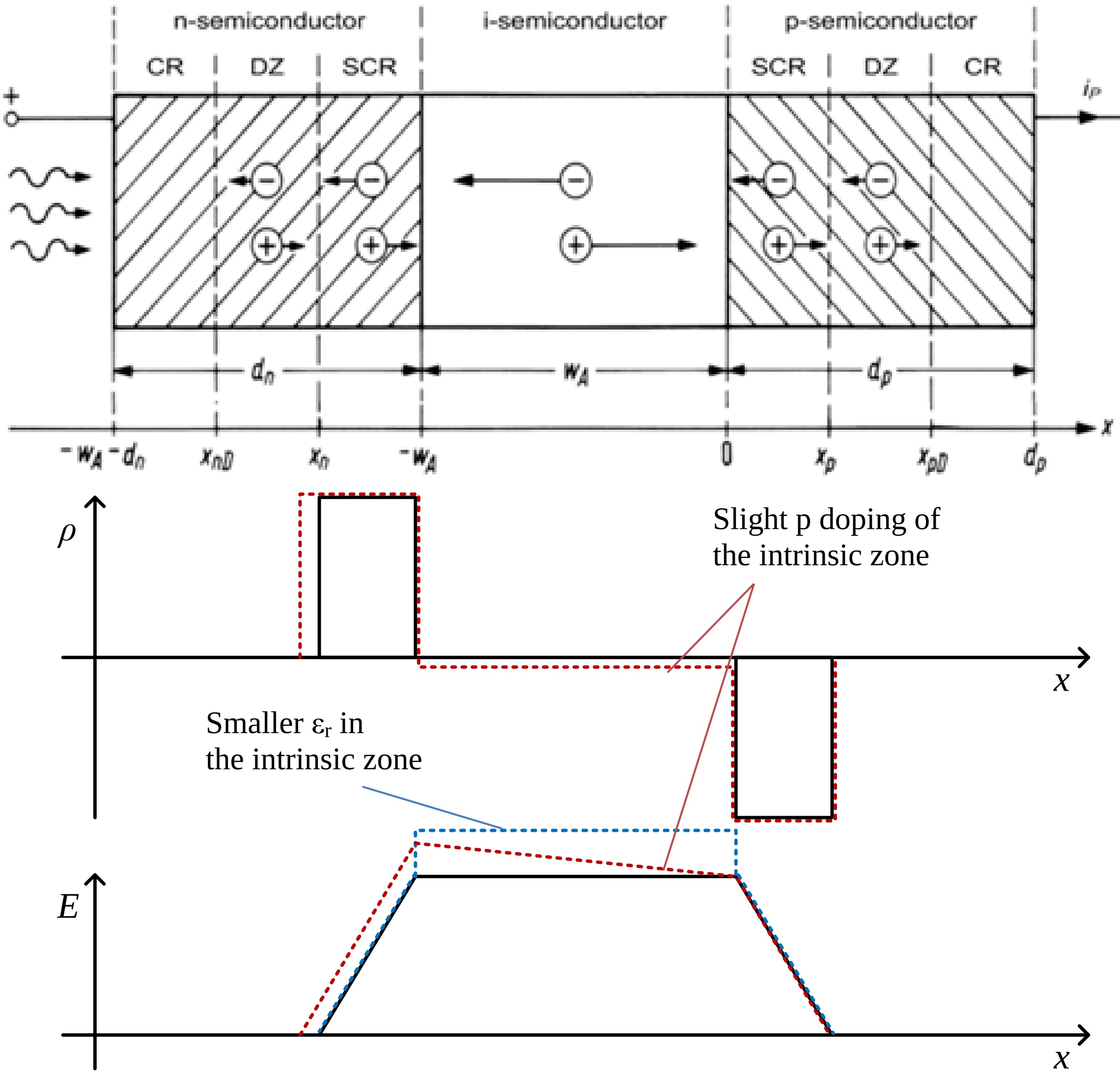

Figure 2: Schematic of a pin-diode, which comprises an intrinsic semiconductor sandwiched by a p- and an n-doped layer. This leads to the formation of the contact region (CR), the diffusion zone (DZ) and the space-charge region (SCR), which is also called depletion region. The width of the n-doped (p-doped) semiconductor is denoted as d_n (d_p) and w_A is the width of the intrinsic absorption zone. The structure is not drawn to scale.

a) In this configuration, the electric field can be expressed as $\vec{E}(x) = E(x)\vec{e}_x$. How is the electric field $E(x)$ related to the space charge distribution $\rho(x)$. Sketch the space charge profile and the electric field as a function of x. The depletion approximation ("Schottky-Näherung") holds. Assume that the dielectric constant ε_r is constant over the device length.

   ➔ See Fig. 2

b) Assume now that the intrinsic zone has a smaller ε_r than the surrounding regions and sketch the corresponding E-field profile.

   ➔ $D = \varepsilon_0\varepsilon_r E$ = const. If ε_r varies there is a jump in the electric field.

c) In practice it is technologically not possible to avoid a slight (p- or n-) doping of the absorption zone. How does this change $E(x)$? Give a qualitative remark, no calculation necessary.

   ➔ The intrinsic zone will have additionally some space charges which tilt the field.

d) From Maxwell's equations it is given that $\nabla \times \vec{H} = \vec{J} + \varepsilon \dfrac{\partial \vec{E}}{\partial t}$. Using this relation and the one-dimensional approximation show that the sum of the transport and displacement currents in the photodiode is independent of x,

$$\frac{\partial}{\partial x}\left(i_n(x,t) + i_p(x,t) + A\varepsilon \frac{\partial E(x,t)}{\partial t} \right) = 0.$$

In this relation the hole and electron currents are defined by $i_p(x,t) = J_p(x,t) \cdot A$, $i_n(x,t) = J_n(x,t) \cdot A$, where A is the cross-sectional area of the diode and where $J_{p,n}$ are the respective current densities. Assume that diffusion currents can be neglected compared to drift currents within the intrinsic region.

➔ Generally it holds $\nabla(\nabla \times \vec{H}) = 0$ the divergence of a curl is always zero. Therefore we can write:

$$\nabla \cdot \left(\vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t} \right) = 0 \quad \text{for the one dimentional case we can write this as}$$

$$\frac{\partial}{\partial x}\left(J_n(x,t) + J_p(x,t) + \varepsilon \frac{\partial E(x,t)}{\partial t} \right) = 0$$

$$\frac{\partial}{\partial x}\left(A \cdot J_n(x,t) + A \cdot J_p(x,t) + A \cdot \varepsilon \frac{\partial E(x,t)}{\partial t} \right) = A \cdot 0$$

$$\frac{\partial}{\partial x}\left(i_n(x,t) + i_p(x,t) + A \cdot \varepsilon \frac{\partial E(x,t)}{\partial t} \right) = 0$$

e) Consider now the case, where the diode is biased with a constant voltage U_B, i.e. $\int_{-w_A}^{0} E(x,t)\mathrm{d}x = U_B = \mathrm{const}$. Using the result from d) show that the total current $i(t) = i_n(x,t) + i_p(x,t) + A\varepsilon \dfrac{\partial E(x,t)}{\partial t}$ depends only on the total number N_n and N_p of electrons and holes that are drifting within the intrinsic zone,

$$i(t) = \frac{e}{\tau_n} N_n(t) + \frac{e}{\tau_p} N_p(t),$$

The quantities τ_n and τ_p are the carrier transit times, $\tau_{n,p} = w_A / v_{n,p}$, where v_n, v_p are the saturation drift velocities of electrons and holes.

➔ With these assumptions we get:

$$\frac{1}{w_A}\int_{-w_A}^{0} i(t)\mathrm{d}x = \frac{1}{w_A}\left(\int_{-w_A}^{0} i_n(x,t)\mathrm{d}x + \int_{-w_A}^{0} i_p(x,t)\mathrm{d}x + \underbrace{\int_{-w_A}^{0} A\varepsilon \frac{\partial E(x,t)}{\partial t}\mathrm{d}x}_{=A\varepsilon \frac{\partial U_B(t)}{\partial t} = 0} \right)$$

$$i(t) = \frac{1}{w_A}\left(\int_{-w_A}^{0} Aen_C(x,t)v_n\mathrm{d}x + \int_{-w_A}^{0} Aep(x,t)v_p\mathrm{d}x \right)$$

$$i(t) = \frac{N_n(t)ev_n}{w_A} + \frac{N_p(t)ev_p}{w_A} = \frac{N_n(t)e}{\tau_n} + \frac{N_p(t)e}{\tau_p}$$

Questions and Comments:

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