9. Tutorial on Optical Sources and Detectors

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Problem 1: Amplitude-phase coupling

Modulating the injection current of a semiconductor laser affects both the real part and the imaginary part of the complex refractive index $\underline{n} = n - \mathrm{j}n_i$ within the active zone. To a first-order approximation, the change of n and n_i are linked by the so-called Henry-factor α_H,

$$\alpha_H = \frac{\partial n / \partial n_c}{\partial n_i / \partial n_c}.$$

The imaginary part of the refractive index is connected to the material gain g and the gain rate G by

$$g = -2k_0 n_i; \quad G = v_g g,$$

where v_g denotes the group velocity of the signal. If the refractive index in the active zone changes by a small amount, then the effective index of the guided mode changes by $\Delta n_e = \Gamma \cdot \Delta n$, where Γ denotes the field confinement factor.

a) How does the change of n influence the emission frequency, when assuming that the laser always emits in the same longitudinal mode? Show that the change $\Delta\omega$ of emission frequency can be expressed by

$$\Delta\omega = -\frac{\omega}{n_{eg}} \Delta n_e,$$

where $n_{eg} = n_e + \omega \frac{\partial n_e}{\partial \omega}$ denotes the effective group refractive index of the mode.

➔ From the resonance condition $n_e \omega = const$, therefore the derivative must be zero

$$\mathrm{d}(n_e\omega) = \frac{\partial(n_e\omega)}{\partial\omega}\mathrm{d}\omega + \frac{\partial(n_e\omega)}{\partial n_e}\mathrm{d}n_e = 0$$

$$= \left(n_e + \omega\frac{\partial n_e}{\partial\omega}\right)\mathrm{d}\omega + \omega\,\mathrm{d}n_e$$

$$= n_{eg}\,\mathrm{d}\omega + \omega\,\mathrm{d}n_e$$

$$\Rightarrow \quad \Delta\omega = -\frac{\omega}{n_{eg}}\Delta n_e$$

b) Calculate the instantaneous deviation $\Delta\omega$ from the steady-state emission frequency as a function of the instantaneous laser gain rate $G(t)$. Start from the definition of the Henry-factor. During the derivation use the solution obtained from part a) and the fact that $\Gamma G_0 = 1/\tau_P$ in steady-state. The resulting equation can be brought to the following form:

$$\Delta\omega = \frac{\alpha_H}{2}\left(\Gamma G - \frac{1}{\tau_P}\right)$$

$$\Rightarrow \quad \Delta(\Gamma G) = \Gamma G - \Gamma G_0 = \Gamma G - \frac{1}{\tau_P}$$

$$\Delta(\Gamma G) = \Gamma\Delta G = \Gamma\Delta(v_g g) = \Gamma v_g \Delta g$$

From the definition of the Henry-factor one can deduce the following relation:

$$\alpha_H = \frac{\Delta n}{\Delta n_i} = -2k_0\frac{\Delta n}{\Delta g}.$$

This can be inserted into the equation above:

$$\Gamma v_g \Delta g = -2k_0 \Gamma v_g \frac{\Delta n}{\alpha_H} = -2k_0 v_g \frac{\Delta n_e}{\alpha_H} = \Gamma G - \frac{1}{\tau_P}.$$

After inserting the relation found in part a) it is possible to calculate

$$\Delta\omega = \frac{\alpha_H}{2}\left(\Gamma G - \frac{1}{\tau_P}\right)$$

c) What are the consequences of this behavior for a time dependent optical signal, which was generated by directly modulating the injection current of a laser?

➔ A time dependent optical signal is generated by varying the laser injection current. This induces also a time dependent gain factor, which then translates into a time dependent change of the instantaneous emission frequency. Hence an intensity modulation comes along with a frequency modulation. This effect is also called a chirp.

Problem 2: Sensitivity of a photodiode

a) A photodiode has a sensitivity of $S = 1\,\mathrm{A/W}$ and is illuminated by an optical signal with power of $-30\,\mathrm{dBm}$. What is the photocurrent generated by the device?

➔ $i_P = S \cdot P = 1\,\mathrm{A/W} \cdot 1\,\mu\mathrm{W} = 1\,\mu\mathrm{A}$

b) The data sheet of a newly developed Si pin photodiode claims a sensitivity of $S = 0.7\,\mathrm{mA/mW}$ at a wavelength of $\lambda = 0.8\,\mu\mathrm{m}$. Comment this statement.

➔ $\eta = \dfrac{Shc}{e\lambda} = 1.085$

This would mean that more electrons are generated than there are photons available, which is a contradiction.

Questions and Comments:

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