8. Tutorial on Optical Sources and Detectors

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Problem 1: Small-signal modulation of a laser diode

In so-called direct modulation schemes, data is encoded on a laser beam modulating the pump current and thereby the laser output power. The question is how fast this modulation can be. This can be investigated by solving the laser rate equations,

$$\frac{\mathrm{d}N_P}{\mathrm{d}t} = N_P \Gamma G\left(n_c, N_P\right) + Q\frac{n_c V}{\tau_{\mathrm{eff}}} - \frac{N_P}{\tau_P}$$

$$\frac{\mathrm{d}\left(n_c V\right)}{\mathrm{d}t} = -N_P \Gamma G\left(n_c, N_P\right) - \frac{n_c V}{\tau_{\mathrm{eff}}} + \frac{I}{e}$$

where N_P denotes the number of photons in the mode under consideration, Γ the confinement factor, $G(n_c, N_P)$ the gain rate, n_c the carrier density, V the active volume, τ_{eff} and τ_P are the effective electron and the photon lifetime respectively. Q is the fraction of spontaneous emission that emits into the mode under consideration. In the general form, the gain rate $G(n_c, N_P)$ depends on the carrier density and on the number of photons; the rate equations are hence nonlinear and cannot be solved analytically. For a basic understanding it is however sufficient to consider a linear small-signal approximation about a stationary operation point, which is given by N_{P0}, n_{c0} and I_0. The time-dependent, small-signal perturbations around the stationary operation point are denoted as $N_{P1}\left(t\right), n_{c1}\left(t\right)$ and $I_1\left(t\right)$.

$$N_P\left(t\right) = N_{P0} + N_{P1}\left(t\right)$$

$$n_c\left(t\right) = n_{c0} + n_{c1}\left(t\right)$$

$$I\left(t\right) = I_0 + I_1\left(t\right)$$

For the small-signal analysis we neglect spontaneous emission and assume a linear gain model without gain compression, i.e. $G\left(n_c, N_P\right) = G_d \cdot \left(n_c - n_{c.tr}\right)$, with the differential gain G_d and the transparency carrier density $n_{c,tr}$. Moreover we assume that the effective carrier lifetime does not depend on the carrier density, i.e. $\tau_{\mathrm{eff}}\left(n_c\right) = \mathrm{const.}$

a) Linearize the laser rate equations by inserting the small-signal ansatz and calculate the transfer function describing the relationship between the photon number $\tilde{N}_{P1}\left(\omega\right)$ and the modulating current $\tilde{I}_1\left(\omega\right)$, where the tilde denotes the Fourier transform of the respective time-dependent function. Bring the solution to the following form:

$$\frac{\tilde{N}_{P1}\left(\omega\right)/\tau_P}{\tilde{I}_1\left(\omega\right)/e} = \frac{\omega_r^2}{-\omega^2 + 2\mathrm{j}\omega\gamma_r + \omega_r^2}$$

where

$$\omega_r^2 = \frac{N_{P0}\Gamma G_d}{V\tau_P}, \text{ and } 2\gamma_r = \frac{1}{\tau_{\mathrm{eff}}} + \tau_P \omega_r^2$$

Hint: Make use of the fact that N_{P0}, n_{c0} and I_0 are stationary-state solutions for which the time derivatives in the rate equations vanish.

➔ Inserting the respective assumptions the rate equations become:

$$(1)\quad \frac{\mathrm{d}N_P}{\mathrm{d}t} = N_P \Gamma G_d \cdot \left(n_c - n_{c,\mathrm{tr}}\right) - \frac{N_P}{\tau_P}$$

$$(2)\quad \frac{\mathrm{d}n_c V}{\mathrm{d}t} = \frac{I}{e} - N_P \Gamma G_d \cdot \left(n_c - n_{c,\mathrm{tr}}\right) - \frac{n_c V}{\tau_{\mathrm{eff}}}$$

Now the small signal ansatz can be inserted. After subtracting the steady-state solution and neglecting products of the small signal quantities:

$$(1')\quad \frac{\mathrm{d}N_{P1}(t)}{\mathrm{d}t} = n_{c1}(t) N_{P0} \Gamma G_d + N_{P1}(t) \overbrace{\left(\Gamma G_d (n_{c0} - n_{c,tr}) - \frac{1}{\tau_P}\right)}^{=\Gamma G_0 - \frac{1}{\tau_P} = 0}$$

$$(2')\quad \frac{\mathrm{d}n_{c1}(t) V}{\mathrm{d}t} = \frac{I_1(t)}{e} - N_{P1}(t) \frac{1}{\tau_P} - n_{c1}(t) \left(N_{P0} \Gamma G_d + \frac{V}{\tau_{\mathrm{eff}}}\right)$$

These coupled differential equations can be solved by Fourier transformation:

$$(1'')\quad \mathrm{j}\omega \tilde{N}_{P1}(\omega) = \tilde{n}_{c1}(\omega) N_{P0} \Gamma G_d$$

$$(2'')\quad j\omega \tilde{n}_{c1}(\omega) V = \frac{\tilde{I}_1(\omega)}{e} - \tilde{N}_{P1}(\omega) \frac{1}{\tau_P} - \tilde{n}_{c1}(\omega) \left(N_{P0} \Gamma G_d + \frac{V}{\tau_{\mathrm{eff}}}\right)$$

Now $\tilde{n}_{c1}(\omega)$ can be eliminated and the resulting equation can be solved to obtain:

$$\frac{\tilde{N}_{P1}(\omega)/\tau_P}{\tilde{I}_1(\omega)/e} = \frac{\dfrac{N_{P0}\Gamma G_d}{V\tau_P}}{-\omega^2 + \mathrm{j}\omega\left(\dfrac{N_{P0}\Gamma G_d}{V} + \dfrac{1}{\tau_{\mathrm{eff}}}\right) + \dfrac{N_{P0}\Gamma G_d}{V\tau_P}} = \frac{\omega_r^2}{-\omega^2 + 2\mathrm{j}\omega\gamma_r + \omega_r^2}$$

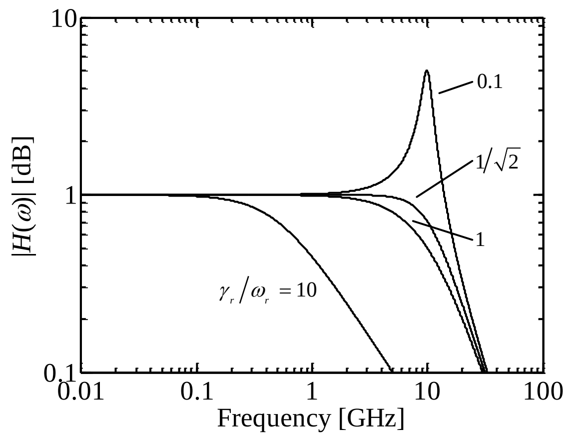

Figure 1 Magnitude of the small signal transfer function for different ratios γ_r/ω_r, where $\omega_r = \mathrm{const.} = 10\,\mathrm{GHz}$.

b) For a given relaxation frequency ω_r, what is the maximum 3dB bandwidth $\omega_{3\mathrm{dB}}^{\max}$ of the laser and for which value of γ_r can it be achieved. The value corresponds to the case of critical damping.
Note: The 3dB bandwidth $\omega_{3\mathrm{dB}}$ is defined as

$$\left|\frac{\tilde{N}_{P1}(\omega_{3\mathrm{dB}})/\tau_P}{\tilde{I}_1(\omega_{3\mathrm{dB}})/e}\right| = \frac{1}{\sqrt{2}}.$$

➔ The magnitude squared of the transfer function can be calculated and set to 1/2:

$$\left|\frac{\tilde{N}_{P1}(\omega_{3\mathrm{dB}})/\tau_P}{\tilde{I}_1(\omega_{3\mathrm{dB}})/e}\right|^2 = \frac{\omega_r^4}{\left(\omega_r^2-\omega_{3\mathrm{dB}}^2\right)^2+4\omega_{3\mathrm{dB}}^2\gamma_r^2} = \frac{1}{2}$$

This equation can be solved for

$$\omega_{3\mathrm{dB}}^2 = \left(\omega_r^2-2\gamma_r^2\right)+\sqrt{\left(\omega_r^2-2\gamma_r^2\right)^2+\omega_r^4}$$

which can then be maximized by solving

$$\frac{\mathrm{d}\omega_{3\mathrm{dB}}^2}{\mathrm{d}\omega_r} = 0$$

leading to $\omega_r^2 = 2\gamma_r^2$, which yields $\omega_{3\mathrm{dB}}^{\max} = \omega_r$.

c) To increase the modulation speed of a laser, the relaxation frequency ω_r should be maximized. Discuss different design approaches that allow to increase ω_r and shortly explain the physical mechanisms behind them.

➔ As derived in part b) the maximum 3dB bandwidth is ω_r and can be achieved in the case of critical damping, i.e. $\gamma_r/\omega_r = 1/\sqrt{2}$. The relaxation frequency can be maximized by three means:

- Small photon lifetime τ_P: This can be achieved by increasing the outcoupling, i.e. small mirror reflectivities. However this also increases the laser threshold current and broadens its linewidth.
- Large photon density N_{P0}/V: That means that the active volume of the laser should be small. When operating the laser a large bias current can be used. However limits are set by temperature issues and the onset of multimode operation.
- Large differential gain ΓG_d: This can be influenced by the doping and strain as well as by special geometries of the active zone, e.g. quantum wells or dots.

Questions and Comments:

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