# 5. Tutorial on Optical Sources and Detectors

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**Problem 1: Spontaneous Emission in an Amplifier**

a) Combining the spontaneous and stimulated emission in a fiber amplifier, the power evolves according to the differential equation from section 2.2.2 of the lecture notes:

$$\frac{\mathrm{d}P(z)}{\mathrm{d}z} = (N_2 - N_1) n \sigma(f_s) P(z) + \xi(f_s)$$

For a fiber section of length dz, the spontaneous emission within the filter bandwidth $B$ is given by $\xi(f_s) = N_2 n_{eg} M_T B \sigma(f_s) hf$ , where $n_{\mathrm{eg}}$ is the effective group refractive index of the fiber and $M_{\mathrm{T}}$ the number of transversal modes.

Show that with a given initial power $P(0)$ this differential equation can be solved to:

$$P(z) = P(0) G_s + (G_s - 1) n_{spon} M_T Bhf$$

Where $G_s = \exp(gL)$ describes the single pass gain and the population inversion is defined as $n_{spon} = \frac{N_2}{N_2 - N_1}$

b) An unsaturated laser amplifier of length $d$ and gain coefficient $g(f)$ amplifies an input signal $P_{\mathrm{s}}(0)$ of the frequency $f$ and introduces amplified spontaneous emission (ASE) at a rate $\xi$ (per unit length). The amplified signal power is $P_{\mathrm{s}}(d)$ and the ASE at the output is $P_{\mathrm{ASE}}$. Sketch the dependence of the ratio $P_{\mathrm{s}}(d)/P_{\mathrm{ASE}}$ on the product of the amplifier gain coefficient and length: $g(f)d$.

**Problem 2: Condition for optical gain in a semiconductor**

It has been shown in the lecture that the photon emission rate is approximately the difference between the rate of absorption and stimulated emission:

$$\frac{dN_p}{dt} \approx r_{st} - r_{ab} = N_p A_{12} \left( n(W_2) p(W_1) - p(W_2) n(W_1) \right),$$

where $n$ ($p$) stands for the number of electrons (holes) at the energy level $W_1$ or $W_2$ respectively and $W_1$ is an energy level within the valence band whereas $W_2$ denotes an energy level within the conduction band. For net optical gain the photon emission rate needs to be positive.

a) In the case of a semiconductor in thermal equilibrium show that this condition cannot be fulfilled!

b) Now consider a semiconductor that has been displaced from thermal equilibrium. However this perturbation is slow enough for the semiconductor to reach a quasi thermal equilibrium with the two quasi Fermi levels $W_{Fn}$ and $W_{Fp}$. Show that it is now possible to obtain net optical gain! Derive the condition that the quasi Fermi levels need to fulfill in order to obtain net optical gain!

c) How can population inversion in a semiconductor be achieved? Name at least two different ways!

**Questions and Comments:**

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