4. Tutorial on Optical Sources and Detectors

May 22nd 2012

Problem 1: Saturation flux density

a) In the general two level atomic system τ_{21} represents the spontaneous emission lifetime of the system, without any stimulated emission. By including a stimulated emission lifetime τ_{20}, the overall emission lifetime τ_2 of the upper level can be described as $1/\tau_2 = 1/\tau_{21}+1/\tau_{20}$. Find the photon-flux density Φ at which the lifetime τ_2 decreases to half of its value. Compare this to the saturation photon-flux density Φ_s.

➔ The rate equation for the second level can be written as:

$$\frac{\mathrm{d}N_2}{\mathrm{d}t} = R_p - N_2 \underbrace{n\sigma(f)\Phi}_{1/\tau_{20}} + N_1 n\sigma(f)\Phi - \frac{N_2}{\tau_{21}}$$

$$\frac{1}{\tau_2} = \left(\frac{1}{\tau_{20}} + \frac{1}{\tau_{21}}\right) = \left(n\sigma(f)\Phi + \frac{1}{\tau_{21}}\right) \stackrel{!}{=} \frac{1}{0.5\cdot\tau_{21}}$$

$$n\sigma(f)\Phi = \frac{2}{\tau_{21}} - \frac{1}{\tau_{21}} = \frac{1}{\tau_{21}}$$

$$\Phi = \frac{1}{n\sigma(f)\tau_{21}}, \quad \text{compared to } \Phi_s = \frac{1}{2n\sigma(f)\tau_{21}}$$

b) Determine the saturation photon flux density $\Phi_s(f_0)$ and the corresponding saturation optical intensity $I_s(f_0)$ for the homogenously broadened Nd^{3+}:YAG transition with the following characteristics:

- Transition wavelength: $\lambda_s = 1064$ nm
- Transition cross section: $\sigma_s = 3 \cdot 10^{-19}$ cm²
- Spontaneous lifetime: $\tau_{21} = 230$ µs
- Refractive index: $n = 1.82$

➔ With the given values the saturation flux density and the saturation intensity can be calculated:

$$\Phi_s(f) = \frac{1}{2n\sigma(f)\tau_{21}} = \frac{1}{2\cdot 1.86\cdot 3\cdot 10^{-19}\,cm^2\cdot 230\mu s} = 3.98\cdot 10^{21}\frac{1}{cm^2\cdot s}$$

$$I_s(f) = hf\cdot\Phi(f) = \frac{hc/\lambda}{2n\sigma(f)\tau_{21}} = 743.6\frac{W}{cm^2}$$

Problem 2: Amplifier gain saturation

Gain is generally a function of the input power. That is obvious since if the gain would not decrease with higher input power it would be possible to generate infinitely large signals. In this exercise an expression for the gain saturation in optical amplifiers shall be derived.

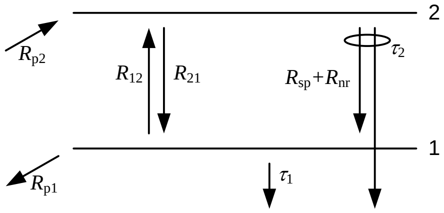


Figure 1 Transition rates between two distinct energy states, including loss and pump rates.

For this case consider a two-level system as depicted in Figure 1. Changes of the carrier density at the higher (2) or lower (1) energy state are modeled by time constants, and external pumping, and internal transition rates. Carriers are injected into the upper energy state and withdrawn from the lower energy state with the respective pump rates R_{p2} and R_{p1}. The carrier depletion by stimulated emission or absorption is given by the rates R_{21} and R_{12}:

$$R_{12} = p_{st} N_1, \quad R_{21} = p_{st} N_2,$$

where $p_{st} = \Phi\sigma$, Φ is the photon flux and σ the transition cross section. Furthermore changes in the carrier density within the system are modeled by time constants that represent the average lifetime of a carrier within the respective state associated with a corresponding loss mechanism. Carriers are depleted from the lower energy state at a rate τ_1 via relaxation of the carriers to lower energy states. From the higher energy state carriers are depleted at a rate τ_2 via spontaneous emission (sp) and non-radiative (nr) transitions, however some of these carriers end up at energy level 1 which is modeled by the recombination rates R_{sp} and R_{nr} for which the following relation holds:

$$R_{sp} + R_{nr} = \frac{N_2}{\tau_{sp}} + \frac{N_2}{\tau_{nr}} = \frac{N_2}{\tau_l}$$

The changes of the carrier densities of the upper and lower energy level, N_2 and N_1, can be described by rate equations:

$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2} - \left(N_2 - N_1\right) p_{st}$$

$$\frac{dN_1}{dt} = -R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_l} + \left(N_2 - N_1\right) p_{st}$$

a) Think about the meaning and origin of each of the individual terms and their respective signs. Assume that the system is in steady state. Calculate the population difference $\Delta N_0 = N_{20} - N_{10}$ in the absence of light, i.e. $\Phi = 0$.

➔

$$0 = R_2 - \frac{N_2}{\tau_2} - \left(N_2 - N_1\right) \cdot 0$$

$$0 = -R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_l} + \left(N_2 - N_1\right) \cdot 0$$

$$N_{20} = R_2 \tau_2$$

$$N_{10} = \left(\frac{N_2}{\tau_l} - R_1\right)\tau_1 = \left(\frac{R_2\tau_2}{\tau_l} - R_1\right)\tau_1$$

$$\Delta N_0 = N_{20} - N_{10} = R_2\tau_2\left(1 - \frac{\tau_1}{\tau_l}\right) + R_1\tau_1$$

b) Now assume a nonzero photon flux Φ and calculate again the difference of carrier densities. Use the result of a) to obtain a solution of the following form:

$$\Delta N = N_2 - N_1 = \frac{\Delta N_0}{1+\Phi/\Phi_s}$$

where the saturation photon flux is given by

$$\Phi_s^{-1} = \left(\tau_2 + \tau_1\left(1-\frac{\tau_2}{\tau_l}\right)\right)\sigma$$

Hint: For the population densities with a non-zero photon flux you should arrive at:

$$N_1 = -\frac{R_2}{p_{st}} + \frac{R_1 - \frac{R_2}{\tau_1 p_{sp}} - R_2}{-\frac{1}{\tau_1\tau_2 p_{st}} - \frac{1}{\tau_1} + \frac{1}{\tau_l} - \frac{1}{\tau_2}}\left(\frac{1}{\tau_2 p_{st}} + 1\right)$$

$$N_2 = \frac{R_1 - \frac{R_2}{\tau_1 p_{sp}} - R_2}{-\frac{1}{\tau_1\tau_2 p_{st}} - \frac{1}{\tau_1} + \frac{1}{\tau_l} - \frac{1}{\tau_2}}$$

$$R_2 - \frac{N_2}{\tau_2} - (N_2 - N_1)p_{st} = 0, \quad (1)$$

➔

$$-R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_l} + (N_2 - N_1) = 0, \quad (2)$$

$$N_1 = \frac{N_2}{\tau_2 p_{st}} - \frac{R_2}{p_{st}} + N_2, \quad (3) \ \rightarrow \text{in (2), solve for } N_2$$

$$\mathrm{N}_2 = \frac{R_1 - \frac{R_2}{\tau_1 p_{sp}} - R_2}{-\frac{1}{\tau_1\tau_2 p_{st}} - \frac{1}{\tau_1} + \frac{1}{\tau_l} - \frac{1}{\tau_2}}, \quad (4)$$

$$\Delta N = N_2 - N_1 = \frac{R_1 - \frac{R_2}{\tau_1 p_{st}} - R_2}{-\frac{1}{\tau_1\tau_2 p_{st}} - \frac{1}{\tau_1} + \frac{1}{\tau_l} - \frac{1}{\tau_2}}\left(1 - 1 - \frac{1}{\tau_2 p_{st}}\right) + \frac{R_2}{p_{st}}$$

$$= \frac{R_1 - \frac{R_2}{\tau_1 p_{st}} - R_2}{\frac{1}{\tau_1} + \frac{\tau_2 p_{st}}{\tau_1} - \frac{\tau_2 p_{st}}{\tau_l} + p_{st}} + \frac{R_2}{p_{st}}, \quad \text{write as one fraction and expand with } \tau_1$$

$$\Delta N = \frac{\overbrace{R_1\tau_1 + R_2\tau_2\left(1-\frac{\tau_1}{\tau_l}\right)}^{\Delta N_0}}{1 + \underbrace{p_{st}}_{\Phi\sigma}\underbrace{\left(\tau_2 + \tau_1\left(1-\frac{\tau_2}{\tau_l}\right)\right)}_{(\Phi_s\sigma)^{-1}}} = \frac{\Delta N_0}{1+\frac{\Phi}{\Phi_s}}$$

c) Calculate the gain g as a function of the photon flux Φ. You should obtain the following relation: $g = \dfrac{g_0}{1+\Phi/\Phi_s}$.

Give the expression for the small signal gain g_0 and create a sketch of g as a function of the photon flux.

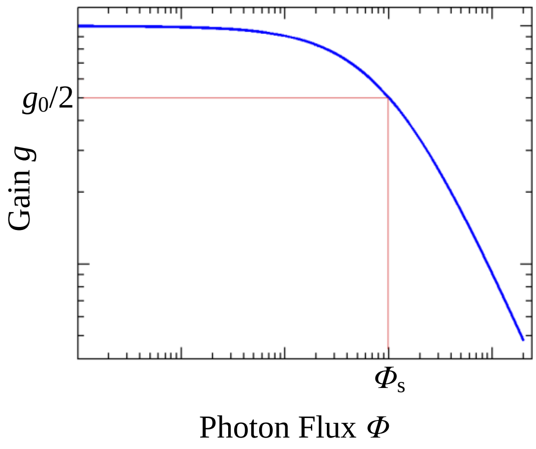
What does that mean for the amplifier output power? Sketch of the amplifier output power as a function of the amplifier input power.

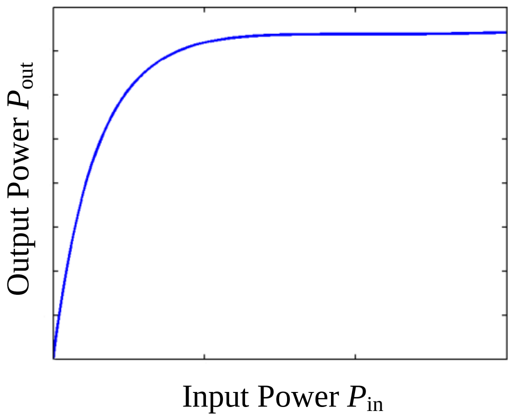
➔
$$g = \Delta N \cdot n \cdot \sigma(f) = \frac{\Delta N_0 \cdot \sigma(f) \cdot n}{1+\dfrac{\Phi}{\Phi_s}}$$

$$g_0 = \Delta N_0 \cdot \sigma(f) \cdot n = \frac{\lambda^2}{8\pi\tau_{sp} n^2} \Delta N_0 \cdot \gamma(f)$$

The small signal gain g_0 can be used when the amplifier is not in saturation; → Φ is much smaller than Φ_s.

The saturation photon flux Φ_s describes the photon flux where the gain reduces to half of its initial value.





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