1. Tutorial on Optical Sources and Detectors

April 24rd 2012

Problem 1: Direct and indirect semiconductors

Figure 1 shows the absorption coefficients of a direct and an indirect semiconductor as a function of wavelength.

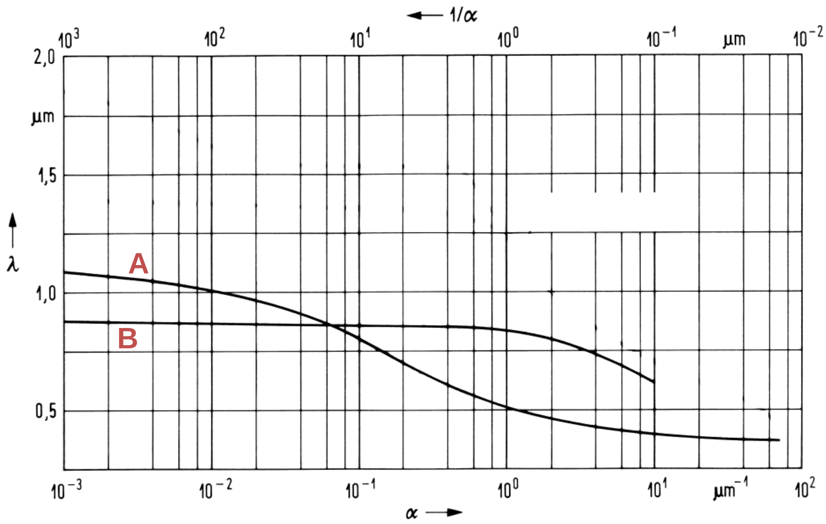

Figure 1: Absorption coefficient of a direct and an indirect semiconductor

a) Which curve belongs to the direct and which one belongs to the indirect semiconductor? Explain your answer.
   - ➔ The absorption coefficient α in curve (A) is increasing slowly for lower wavelengths λ (higher photon energy). This means the absorption mechanism is quite ineffective over a longer wavelength range which is a strong indication for an indirect semiconductor.

     In comparison for curve (B) the absorption coefficient α reaches after a certain wavelength almost instantaneous high values which hints for an effective absorption mechanism, typical for direct band gap semiconductors.

b) What is the band gap energy of both semiconductor materials? Which material could it be?
   - ➔ The absorption in curve (A) starts at $\lambda_a = 1.1$ µm which corresponds to a photon energy of $W_{phot} = h\ f = h\ c/\lambda_a = 1.13$ eV. A typical material with this band gap energy and an indirect band gap is silicon.
   - ➔ For curve (B) the absorption wavelength is around $\lambda_a = 0.85$ µm, which corresponds to a photon energy of $W_{phot} = h\ c/\lambda_a = 1.46$ eV. A direct band gap material with this energy is gallium arsenide.

Figure 2 shows the schematic band diagram of an indirect semiconductor. Holes in the valence band tend towards the energy maximum of the valence band whereas electrons in the conduction band tend towards the conduction band minimum. For the following calculations use the values of silicon: Indirect band-gap energy W_G = 1.13eV and lattice constant a = 0.543nm.

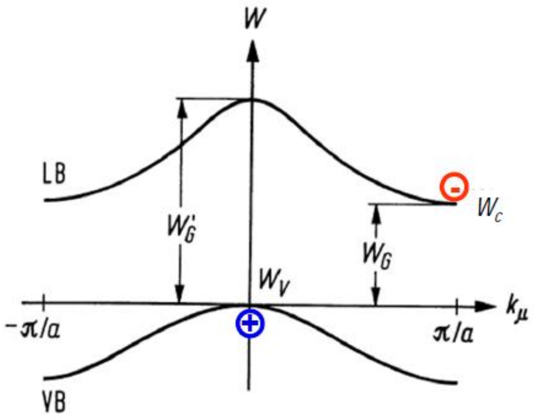

Figure 2: Schematic band diagram of an indirect semiconductor; LB ("Leitungsband") denotes the conduction band; VB is the valence band.

c) Calculate the frequency of a photon that corresponds to the band gap energy of silicon. How large is the momentum of this photon? Compare this momentum to the crystal momentum of the electron.

➔ The momentum of the electron at $k = \pi/a$ is three magnitudes larger than the momentum of the photon at the band gap energy. For absorption the conservation of energy and impulse must be fulfilled, but a photon can only supply the energy but not the momentum to the electron.

$$W_{photon} = h \cdot f, \quad f = \frac{W_{photon}}{h} = 273.2\,\text{THz} \rightarrow \lambda = 1.1\,\mu\text{m}$$

$$p_{photon} = \hbar k = \hbar \frac{2\pi}{\lambda} \approx 6.0 \cdot 10^{-28}\, \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

$$p_{electron} = \hbar k = \hbar \frac{\pi}{a} \approx 6.1 \cdot 10^{-25}\, \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

d) Consider now a phonon with a momentum that corresponds to the crystal momentum of the electron. Compare the energy of the phonon to the band gap energy when you consider a value of v_s = 8433m/s for the speed of sound in silicon.

➔ The energy of the phonon is two orders of magnitude smaller than the band gap energy. The phonon can contribute the momentum to an absorption process, but not the energy.

$$p_{electron} = p_{phonon} = \hbar \cdot k = \hbar \frac{\omega}{v_s},$$

$$W_{phonon} = \hbar\omega = p_{phonon} \cdot v_s = 6.1 \cdot 10^{-25}\, \frac{kg\cdot m}{s} \cdot 8433 \frac{m}{s} \approx 0.032\,\text{eV}$$

$$\text{W}_{photon,gap} = 1.1\text{eV}$$

e) Is it possible to build an efficient light source using silicon? Explain your answer.

➔ Silicon needs a three particle process for light emission. Electrons relaxing from the conduction band minimum to the valence band maximum need in addition to the photon for energy conservation a phonon for momentum conservation. This is not a very likely process, so the emission probability is greatly reduced.

Problem 2: Linear and logarithmic scale conversion

The logarithmic scale is widely used in optical communication. Especially the ratio between two power levels is often described in the dimensionless unit decibel (dB). Such a ratio between power P_1 and power P_0 is normally defined in dB as $10 \cdot \log_{10}(P_1/P_0)$.

a) Calculate the corresponding numbers in dB for the different power ratios of $P_1/P_0 = 1$, 2, 4, 10, 20, 100, 0.5, 0.25, 0.1

➔

linear	1	2	4	10	20	100	0.5	0.25	0.1
dB	0	3	6	10	13	20	-3	-6	-10

b) In contrast to dB, the absolute unit dBm is used to express an absolute power. The unit dBm refers to the power P_1 referenced to 1mW and is therefore defined as $P_{[dBm]} = 10 \cdot \log_{10}(P_1/1\text{mW})$. Fill the following table, use the results from a) and try to do it without a calculator.

linear	**logarithmic**
100 nW	-40 dBm
8 mW	9 dBm
25 µW	-16 dBm
10 mW	10 dBm
2 mW	3 dBm
400 mW	26 dBm
0.2 mW	-7 dBm
5 µW	-23 dBm

Bonus Program:

At three randomly chosen exercises we will collect your notes before the exercise starts. They will be marked and if you accomplish an average of 70% or more of all collected exercises, your oral examination grade will be upgraded by 0.3 or 0.4 (except grades of 1.0 and 4.7 or worse). If you cannot join an exercise, you may also hand your notes to the teaching assistants (see contact details below) before the respective exercise.

Questions and Comments:

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