An Easy Introduction to Plasmonics

Wolfgang Freude

Institute of Photonics and Quantum Electronics (IPQ), University of Karlsruhe, Germany

Universität Karlsruhe (TH)
Institut für Photonik und Quantenelektronik (IPQ)
http://www.ipq.uni-karlsruhe.de

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The Beginning of All

- Maxwell´s equations
- Transverse waves
- Longitudinal waves

Maxwell´s Equations

$$\text{curl}\,\vec{H} = \frac{\partial \vec{D}}{\partial t}\,, \qquad \text{curl}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}\,,$$

$$\text{div}\,\vec{D} = 0\,, \qquad \vec{D} = \epsilon_0 \vec{E} + \vec{P} \qquad \text{div}\,\vec{B} = 0\,, \qquad \vec{B} = \mu_0 \vec{H}$$

Decoupled form:

$$\left(\text{curl}\left(\frac{1}{\epsilon_r}\,\text{curl}\right)\right)\vec{H} = \left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{H}, \qquad \left(\frac{1}{\epsilon_r}\,\text{curl}\,\text{curl}\right)\vec{E} = \left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E}$$

Constitutive equations (in a homogenous dielectric medium, no space charges):

$$\text{div}\,\vec{B} = 0\,, \qquad \text{div}\left(\epsilon_0\epsilon_r\vec{E}\right) = \epsilon_0\epsilon_r\,\text{div}\,\vec{E} + \text{grad}\left(\epsilon_0\epsilon_r\right)\cdot\vec{E} = 0$$

Homog. medium and vector identity $\nabla^2\vec{E} = \text{grad}\,\text{div}\,\vec{E} - \text{curl}\,\text{curl}\,\vec{E}$:

$$\text{grad}\,\text{div}\,\vec{E} - \nabla^2\vec{E} = \left(-\frac{\epsilon_r}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E}$$

Formal Wave Solutions without External Excitation

Wave equation in time domain, homogeneous medium:

$$\text{grad}\,\text{div}\,\vec{E} - \nabla^2 \vec{E} = \left(-\frac{\epsilon_r}{c^2} \frac{\partial^2}{\partial t^2} \right) \vec{E}$$

Plane wave ansatz (plane phase surfaces $\vec{k} \cdot \vec{r} = \text{const}$, $\vec{k} \cdot \vec{k} = k^2$):

$$\vec{E}(t, \vec{r}\,) = \tilde{\vec{E}}(f, \vec{\kappa}\,)\, \mathrm{e}^{\mathrm{j}(\omega t - \vec{k} \cdot \vec{r}\,)}, \; \omega = 2\pi f, \; \vec{k} = 2\pi \vec{\kappa} = k_x\, \vec{e}_x + k_y\, \vec{e}_y + k_z\, \vec{e}_z$$

Operator correspondences ($q = x, y, z$):

$$\partial_t = \mathrm{j}\,\omega\,, \quad \partial_q = -\mathrm{j}\,k_q\,, \quad \text{grad} = -\mathrm{j}\,\vec{k}\,, \quad \text{div} = -\mathrm{j}\,\vec{k} \cdot\,, \quad \nabla^2 = -k^2$$

Wave equation in Fourier domain (Helmholtz equation):

$$-\mathrm{j}\,\vec{k}\left(-\mathrm{j}\,\vec{k} \cdot \tilde{\vec{E}}\right) + k^2 \tilde{\vec{E}} = \epsilon_r \frac{\omega^2}{c^2}\, \tilde{\vec{E}}$$

Transverse waves $\vec{k} \perp \vec{E}$, $\vec{k} \cdot \vec{E} = 0$, dispersion relation:

$$k^2 = \epsilon_r(f, \vec{\kappa}\,) \frac{\omega^2}{c^2}\,, \qquad k_0 = \frac{\omega}{c} \quad \text{(dispersion in vacuum)}$$

Longitudinal waves $\vec{k} \parallel \vec{E}$, $\vec{k}(\vec{k} \cdot \vec{E}) = k^2 \vec{E}$, dispersion relation:

$$\epsilon_r(f, \vec{\kappa}\,) = 0 \qquad \text{for all } \vec{\kappa} \parallel \vec{E}$$

Longitudinal (Compressional) Waves

Even for charge fluctuations in a plasma, the overall system remains electrostatically neutral. Field arises solely from displacement of charges having opposite sign, but occuring in equal numbers.

There is no overall free charge $\Rightarrow \rho = 0$:

$$\epsilon_0 \epsilon_r \,\mathrm{div}\, \vec{E} = 0$$

Polarisation $\vec{P}$ exists, because free carriers may be moved leading to re-radiation (without changing overall charge neutrality!). Therefore also the electric field $\vec{E} \parallel \vec{P}$ exists:

$$\mathrm{div}\, \vec{E} \neq 0 \quad \Rightarrow \quad \epsilon_r = 0$$

$\epsilon_r(\omega_p) = 0$ becomes the condition for plasma oscillations at the plasma eigen frequency $\omega = \omega_p$, i. e., for natural oscillation of a neutral plasma.

Excitation of plasma oscillations only with particles (e. g., electrons traversing a metal foil can loose energy in quanta $\hbar\omega_p$), no coupling to electromagnetic field, no propagation $\mathrm{e}^{\mathrm{j}(\omega t - \sqrt{\epsilon_r}\, k_0 z)}$ with $\epsilon_r = 0$.

Why Does a Local Charge Perturbation Not Propagate?

Poisson equation for a electron concentration perturbation $N_s - N$ compared to the unperturbed concentration N, assuming Boltzmann statistics $N_s = N\,\mathrm{e}^{\varphi/U_T}$, resulting in a small potential perturbation $\varphi/U_T \ll 1$, $U_T = \frac{kT}{e}$:

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}x^2} = -e\,\frac{N_s - N}{\varepsilon} \approx \frac{Ne}{\varepsilon}\left(1 - \mathrm{e}^{\varphi(x)/U_T}\right) \approx \frac{Ne}{\varepsilon U_T}\,\varphi$$

Solution has structure $\mathrm{e}^{\pm x/L_D}$. Proper choice of sign for $x \geq 0$:

$$\varphi(x) = \varphi_O\,\mathrm{e}^{-x/L_D}, \qquad L_D = \sqrt{\frac{\varepsilon U_T}{Ne}}$$

Debye length: Screening length of charge perturbation.

Perturbations of the potential compensated by re-arrangement of carriers inside a distance L_D.

If potential changes inside L_D by $\Delta\varphi \sim U_T$:

$$N \rightarrow N_s = 2.72\,N \qquad \text{for} \qquad N \exp\left(\frac{0}{U_T}\right) \rightarrow N_s = N \exp\left(\frac{U_T}{U_T}\right)$$

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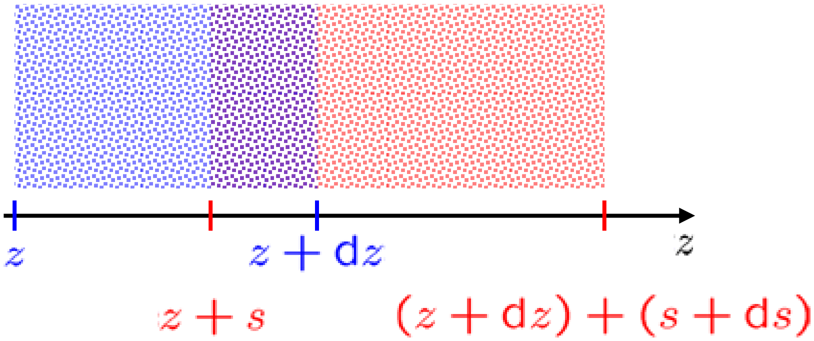
- Plasma density oscillation
  - Electron density perturbation
  - Plasma frequency and dispersion diagram
- Modeling the dielectric constant
  - Bound and free charge carriers
  - Free carriers and dielectric constant
- Dielectric-dielectric interface
  - Boundary conditions
  - Fresnel's formulae. Brewster's angle
- Dielectric-metal interface
  - Boundary conditions. Surface plasmon polariton (SPP)
  - Dispersion diagram. Characteristic lengths
  - SSP excitation. SNOM visualization
- Summary

Plasma Density Oscillation for Local Concentration Perturbation



Electrons in $z \ldots z+\Delta z$ at time t shifted by small amount $s(t,z) \rightarrow$ region $z+s \ldots (z+\mathrm{d}z)+(s+\mathrm{d}s)$

Concentration $N \rightarrow N_s$, $\mathrm{d}s \ll \mathrm{d}z$:

$$N_s = N\frac{\mathrm{d}z}{\mathrm{d}z+\mathrm{d}s} \approx N\left(1-\frac{\mathrm{d}s}{\mathrm{d}z}\right)$$

Background of positive ions with concentration N does not move.

Average charge concentration and electric field $\mathrm{div}\,\vec{E} = \rho/\epsilon_0$:

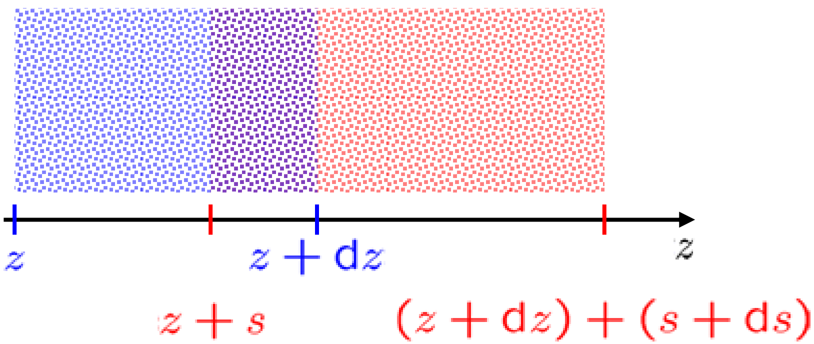
$$\rho = -N_s e + Ne = Ne\frac{\mathrm{d}s}{\mathrm{d}z}, \qquad \frac{\mathrm{d}E_z}{\mathrm{d}z} = \frac{Ne}{\epsilon_0}\frac{\mathrm{d}s}{\mathrm{d}z} \quad \Rightarrow \quad E_z = \frac{Ne}{\epsilon_0}s$$

Other approach: Cuboid of electrons with cross-section A shifted by s ($\mathrm{d}s = 0$) $\rightarrow$ "capacitor" $C = \frac{\epsilon_0 A}{\mathrm{d}z}$ with "electrode" distance $\mathrm{d}z$ and charge $Q = NeAs$ $\rightarrow$ electric field amplitude $E_z = \frac{Q}{C\,\mathrm{d}z} = \frac{Ne}{\epsilon_0}s$

Rest. force $F_z = -eE_z = -\frac{Ne^2}{\epsilon_0}s = -Ks = -m_e\omega_p^2 s \mathrel{\widehat{=}} m_e \times$ accel.:

$$-m_e\omega_p^2 s = m_e\frac{\mathrm{d}^2 s}{\mathrm{d}t^2}, \qquad \omega_p^2 = \frac{K}{m_e} = \frac{Ne^2}{\epsilon_0 m_e} \quad \Rightarrow \quad s(t) = \hat{s}\cos(\omega_p t)$$

Plasma Density Oscillation (Plasmon). Dispersion Diagram



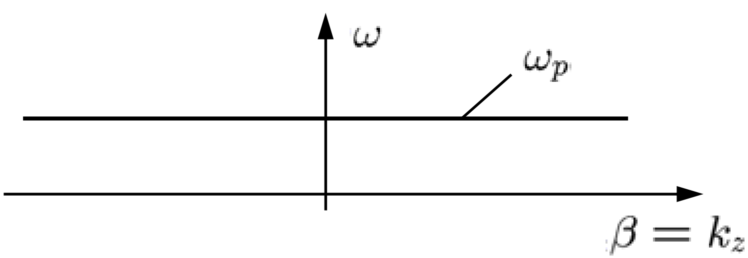
Local density perturbation → electrons oscillate with plasma frequency → $s(t) = \hat{s}\cos(\omega_p t)$:

$$\omega_p^2 = \frac{Ne^2}{\epsilon_0 m_e}$$

Any local perturbation of the electron density results in an undamped oscillation (plasma assumed to be collision-free, no losses).

Adjacent perturbations do not influence each other, so electron density perturbations do not propagate.

Depending on the periodicity of a spatial perturbation (set from outside), any "wavelength" ("propagation constant" $\beta = k_z$) may be chosen ⇒ dispersion diagram for $E_z = \frac{Ne}{\epsilon_0} s$ is a horizontal line.



Arbitr. phase vel., gr. vel. $\frac{\mathrm{d}\omega}{\mathrm{d}\beta} = 0$

No excitation by transverse elmag waves, only by particle impact

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Medium Properties

$$\vec{P}(t,\vec{r}) = \epsilon_0 \int_0^\infty \chi(\tau,\vec{r})\,\vec{E}(t-\tau,\vec{r})\,\mathrm{d}\tau\,,$$

$$\vec{P}(f) = \epsilon_0 \underline{\chi}(f)\vec{E}(f), \qquad \underline{\chi}(f) = \int_0^\infty \chi(t)\,\mathrm{e}^{-\mathrm{j}\,2\pi f t}\,\mathrm{d}t\,,$$

$$\underline{\chi}(f) = \chi(f) + \mathrm{j}\,\chi_i(f) = \epsilon_r(f) - 1 - \mathrm{j}\,\epsilon_{ri}(f), \quad \underline{\chi}(f) = \underline{\chi}^*(-f)$$

Refractive Index and Dielectric Constant $\bar{\epsilon}_r = \bar{n}^2$

$$\bar{n} = n - \mathrm{j}\,n_i, \qquad \bar{\epsilon}_r = \epsilon_r - \mathrm{j}\,\epsilon_{ri},$$

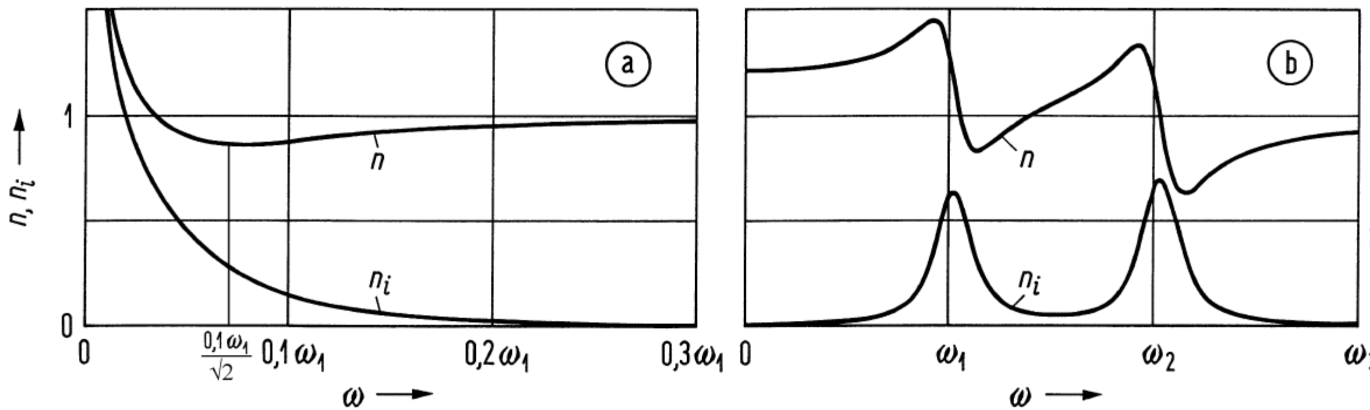
$$\epsilon_r = n^2 - n_i^2, \qquad \epsilon_{ri} = 2nn_i,$$

$$n^2 = \tfrac{1}{2}\epsilon_r\left(1 + \sqrt{1 + \epsilon_{ri}^2/\epsilon_r^2}\right), \qquad n_i = \epsilon_{ri}/(2n),$$

$$n \approx \sqrt{\epsilon_r} \quad (\text{for } |\epsilon_{ri}| \ll \epsilon_r) \quad n_i \approx \epsilon_{ri}/(2\sqrt{\epsilon_r}\,),$$

$$n \approx \sqrt{|\epsilon_{ri}|/2} \quad (\text{for } |\epsilon_{ri}| \gg \epsilon_r) \quad n_i \approx \mathrm{sgn}(\epsilon_{ri})\sqrt{|\epsilon_{ri}|/2}$$

Free and Bound Charge Carriers



$$\vec{D} = \epsilon_0 \epsilon_r \vec{E}$$
$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$
$$\vec{D} = \epsilon_0 (1 + \chi) \vec{E}$$

Fig. 2.1. Real part n und negative imaginary part (n_i) of complex refractive index $\bar{n} = n - \mathrm{j}\, n_i$: Frequency dependence (a) Free carriers only (b) Two collectives of bound charges with high mass (ions, low angular resonance frequency ω_1) and low mass (electrons, high angular resonance frequency ω_2)

Transverse $\vec{E}$-field forces molecules and bound electrons to vibrate. Force $\vec{F}$ exerted on charge q is $\vec{F} = q\vec{E}$. Assuming $\vec{E} = E_x\, \vec{e}_x$, no losses $\rightarrow$ Newton's 2nd law: forces (driving + restoring force $\sim \omega_r^2$) $\widehat{=}\; m_r \times$ acceleration $\rightarrow$ electric dipole moment $\vec{P} = qN\,\vec{x}$:

$$q\hat{E}_x \cos\omega t - m_r \omega_r^2 x = m_r \frac{\mathrm{d}^2 x}{\mathrm{d}t^2}, \qquad \vec{x}(f) = \frac{q/m_r}{\omega_r^2 - \omega^2}\,\vec{E}(f) = \frac{\vec{P}(f)}{qN}$$

$$\bar{\epsilon}_r = \bar{n}^2 = 1 + \frac{N\,e^2}{\epsilon_0 m_r}\,\frac{1}{\omega_r^2 - \omega^2}$$

Free Carriers and Dielectric Constant

Spatially fixed positive ions with concentration N providing f_e free electrons each, which produce dielectric constant $\bar{\epsilon}_r = \epsilon_r - \mathrm{j}\,\epsilon_{ri}$.

Transverse $\vec{E}$-field makes free electrons vibrate in unison $\rightarrow$ No density perturbation $\rightarrow$ No restoring force $\Rightarrow$ $\omega_r = 0$. Electron displacement $\vec{x} \sim -q\vec{E}$ opposite to driving force $\vec{F} = q\vec{E}$ (no loss: 180° phase *at all frequencies* $\omega \neq 0$):

$$\text{Bound:} \quad \epsilon_r = n^2 - n_i^2 = 1 + \frac{Ne^2}{\epsilon_0 m_r} \frac{1}{\omega_r^2 - \omega^2}, \quad \epsilon_{ri} = 2nn_i = 0$$

$$\text{Free:} \quad \epsilon_r = n^2 - n_i^2 = 1 - \frac{N f_e e^2}{\epsilon_0 m_e} \frac{1}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}, \qquad \omega_p^2 = \frac{N f_e e^2}{\epsilon_0 m_e}$$

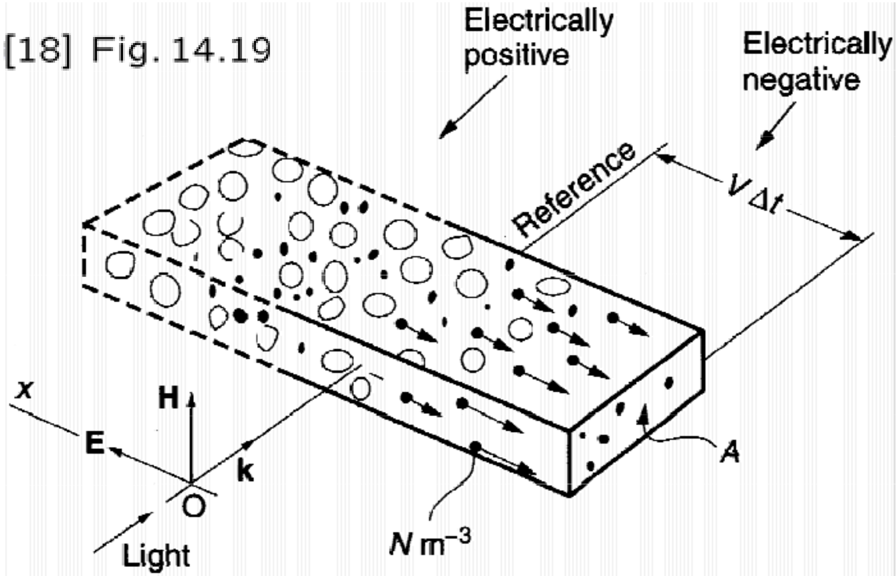
Free electrons oscillating out of phase with incident light, re-radiate wavelets tending to cancel incoming disturbance: $\Rightarrow 0 < \epsilon_r < 1$

At $\omega = \omega_p$, no plasma resonance is excited, but re-radiated wavelets cancel incoming disturbance exactly: $\Rightarrow \epsilon_r = 0$

For $\omega \leq \omega_p$, no propagation: $\Rightarrow \mathrm{e}^{\mathrm{j}(\omega t - \sqrt{\epsilon_r}\, k_0 z)} = \mathrm{e}^{-\sqrt{|\epsilon_r|}\, k_0 z}\, \mathrm{e}^{\mathrm{j}\,\omega t}$

Free Carriers and Dielectric Constant: Yet Another Approach

[18] Fig. 14.19



Transverse $\vec{E}$-field forces free electrons to vibrate. Now with velocity $\vec{v}(t) = \hat{v}_x \, \mathrm{e}^{\mathrm{j}\,\omega t} \, \vec{e}_x$:

$$-e\hat{E}_x \, \mathrm{e}^{\mathrm{j}\,\omega t} = -m_e \frac{\mathrm{d}}{\mathrm{d}t}\left(\hat{v}_x \, \mathrm{e}^{\mathrm{j}\,\omega t}\right)$$

$$\hat{v}_x(f) = -\mathrm{j}\,\frac{e}{m_e\,\omega}\,\hat{E}_x(f)$$

Total charge passing A:

$$\mathrm{d}Q = eNAv_x \, \mathrm{d}t$$

Plasma current density:

$$J_x = \frac{1}{A}\frac{\mathrm{d}Q}{\mathrm{d}t} \quad \Rightarrow \quad \hat{J}_x(f) = -\mathrm{j}\,\frac{Ne^2}{m_e\,\omega}\,\hat{E}_x(f) \;\rightarrow\; \vec{J}(f) = -\mathrm{j}\,\frac{Ne^2}{m_e\,\omega}\,\vec{E}(f)$$

Maxwell, this time with $\mathrm{curl}\,\vec{H} = \frac{\mathrm{d}\vec{D}}{\mathrm{d}t} + \vec{J}$ and $\vec{D} = \epsilon_0 \vec{E}$:

$$\mathrm{curl}\,\vec{H}(f) = \mathrm{j}\,\omega\epsilon_0\vec{E}(f) + \vec{J}(f) = \mathrm{j}\,\omega\epsilon_0\left(1 - \frac{Ne^2}{\epsilon_0 m_e}\frac{1}{\omega^2}\right)\vec{E}(f)$$

Result: $\bar{\epsilon}_r = \bar{n}^2 = 1 - \frac{\omega_p^2}{\omega^2}$, $\omega_p^2 = \frac{Ne^2}{\epsilon_0 m_e}$. Semicond.: Background-$\epsilon_r$!

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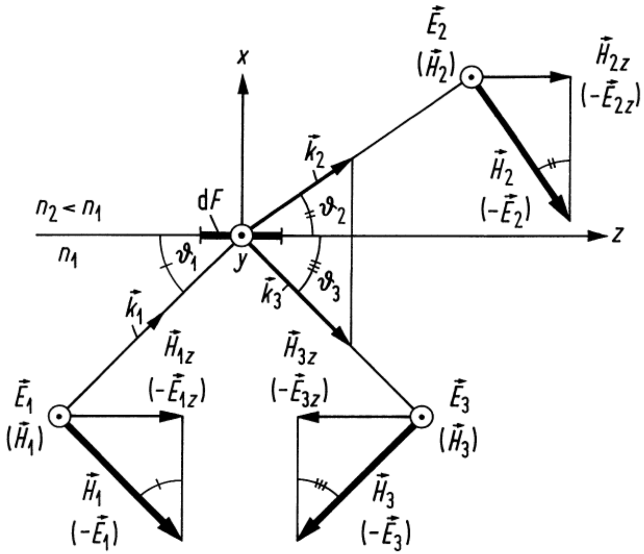
Dielectric-Dielectric Interface — Boundary Conditions



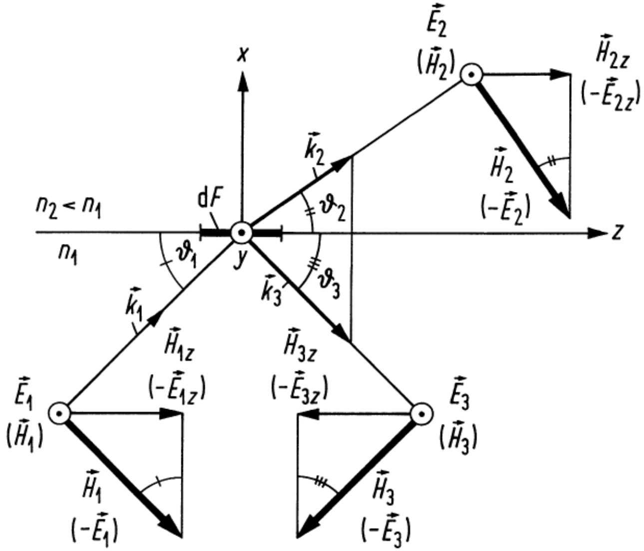
$$\vec{\Psi}(t,\vec{r}) = \vec{\Psi}\exp(\mathrm{j}\,\omega t - \mathrm{j}\,\vec{k}_s\cdot\vec{r}\,),$$
$$\vec{\Psi}_s = \vec{E}_s, \vec{H}_s\,, \qquad s = 1,2,3\,.$$

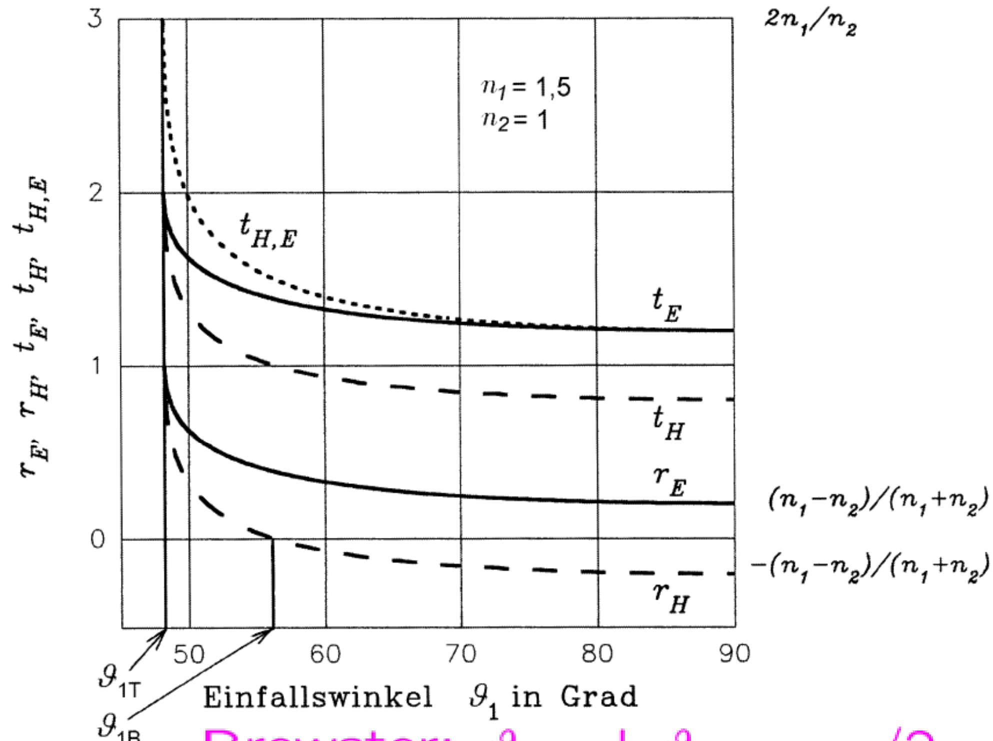
Subscripts 1, 2, 3: Incident, transmitted and reflected waves. Only the total of three solve the problem. Vector component subscripts are coordinates $q = x, y, z$. For the incident wave we assume $k_{1y} = 0$.

E-pol. ($\vec{E}_s = E_s\vec{e}_y \parallel$ boundary plane, $E_{sz} = 0$, TE- or H-wave), or
H-pol. ($\vec{H}_s = H_s\vec{e}_y \parallel$ boundary plane, $H_{sz} = 0$, TM- or E-wave).
Example for transverse E- and H-components in boundary $x = 0$:

$$E_{2y}\exp[-\mathrm{j}(\overbrace{k_{2x}x}^{x=0} + \overbrace{k_{2y}y}^{\to 0} + k_{2z}z)] = E_{1y}\exp[-\mathrm{j}(\underbrace{k_{1x}x}_{x=0} + \underbrace{k_{1y}y}_{=0} + k_{1z}z)]$$
$$+\,E_{3y}\exp[-\mathrm{j}(\underbrace{k_{3x}x}_{x=0} + \underbrace{k_{3y}y}_{\to 0} + k_{3z}z)]$$

Dielectric-Dielectric Interface — Fresnel´s Formulae





Always $\vartheta_1 > \vartheta_2$ if $n_2 < n_1$.

At $\vartheta_1 = \vartheta_{1T} \rightarrow \vartheta_2 = 0$.

Brewster: $\vartheta_{1B} + \vartheta_{2B} = \pi/2$

Fresnel's formulae:

Amplitude reflection and transmission coefficients as a function of incident angle ϑ_1 for $n_1 = 1.5$, $n_2 = 1$. (glass-air interface). Einfallswinkel ϑ_1 in Grad = incident angle ϑ_1 in degree

$$r_E = \frac{E_3}{E_1} = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{k_{1x} - k_{2x}}{k_{1x} + k_{2x}} = \frac{n_1 \sin\vartheta_1 - n_2 \sin\vartheta_2}{n_1 \sin\vartheta_1 + n_2 \sin\vartheta_2},$$

$$r_H = \frac{H_3}{H_1} = \frac{Y_2 - Y_1}{Y_2 + Y_1} = \frac{n_2^2 k_{1x} - n_1^2 k_{2x}}{n_2^2 k_{1x} + n_1^2 k_{2x}} = \frac{n_2 \sin\vartheta_1 - n_1 \sin\vartheta_2}{n_2 \sin\vartheta_1 + n_1 \sin\vartheta_2}$$

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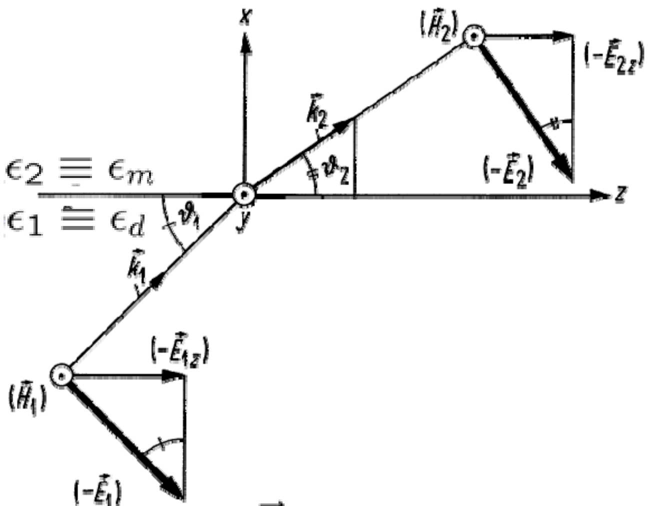
Dielectric-Metal Interface

Wanted: A wave that is (in x-direction) confined to the surface and travels along the z-axis.

- Generally, all three waves (incident, transmitted, reflected) propagate along x (and z).
- For TIR it is known that transmitted wave is evanescent, and that only incident and reflected waves propagate.
- With a total of three plane waves (and only one boundary) no better confinement is to be expected:
  - If incident wave exists, there must be also a reflected wave.
  - In H-polarisation and at the Brewster angle, there are only two waves, an “incident” and a “transmitted” wave.
- Try ansatz with H-polarisation and two plane waves only!

Dielectric-Metal Interface — Plane Wave Ansatz



$$\vec{\Psi}(t, \vec{r}\,) = \vec{\Psi} \exp(\mathrm{j}\,\omega t - \mathrm{j}\,\vec{k}_s \cdot \vec{r}\,),$$

$$\vec{\Psi}_s = \vec{E}_s, \vec{H}_s\,, \qquad s = 1\,(d), 2\,(m)$$

Subscripts $1\,(d), 2\,(m)$: "Incident", "transmitted". Total of two solves TM problem for Brewster angle. Comp. subscr. by coord. $q = x, y, z$ For incident wave $k_{dy} = 0$.

H-pol. ($\vec{H}_s = H_s \vec{e}_y \parallel$ boundary plane, $H_{sz} = 0$, TM- or E-wave).

H-field continuous at $x = 0 \Rightarrow k_{dz} = k_{mz} = \beta$ and $H_{dy} = H_{my}$:

$$H_{dy} \exp[-\mathrm{j}(\overbrace{k_{dx}x}^{x\,=\,0} + \overbrace{k_{dy}y}^{=\,0} + k_{dz}z)] = H_{my} \exp[-\mathrm{j}(\overbrace{k_{mx}x}^{x\,=\,0} + \overbrace{k_{my}y}^{=\,0} + k_{mz}z)]$$

Ansatz for $H_y(x, z)$ substituted in $\operatorname{curl} \vec{H}(t, \vec{r}\,) = \epsilon_0 \epsilon_r \frac{\partial \vec{E}(t,\vec{r}\,)}{\partial t}$:

$$H_y(x, z) = H_y \exp[-\mathrm{j}(k_x x + \beta z)], \quad \left.\begin{matrix} -\partial_z H_y(x, z) \\ \partial_x H_y(x, z) \end{matrix}\right\} = \mathrm{j}\,\omega \epsilon_0 \epsilon_r \left\{\begin{matrix} E_x(x, z) \\ E_z(x, z) \end{matrix}\right.$$

$$E_z(x, z) = \frac{-\mathrm{j}\,k_x}{\mathrm{j}\,\omega \epsilon_0 \epsilon_r} H_y(x, z), \qquad E_x(x, z) = -\frac{-\mathrm{j}\,\beta}{\mathrm{j}\,\omega \epsilon_0 \epsilon_r} H_y(x, z)$$

Dielectric-Metal Interface — Boundary Conditions

Amplitude continuity for $H_y(0,z)$, $E_z(0,z)$ $(k_{dz} = k_{mz} = \beta)$:

$$\left.\begin{aligned} H_{dy} - H_{my} &= 0 \\ \frac{k_{dx}}{\epsilon_d} H_{dy} - \frac{k_{mx}}{\epsilon_m} H_{my} &= 0 \end{aligned}\right\} \Rightarrow \frac{k_{dx}}{\epsilon_d} - \frac{k_{mx}}{\epsilon_m} = 0 \quad \Rightarrow$$

Brewster: $\epsilon_m k_{dx} = \epsilon_d k_{mx}$

Propagation constant of plane waves in both media:

$$\epsilon_d k_0^2 = k_{dx}^2 + \beta^2, \qquad \epsilon_m k_0^2 = k_{mx}^2 + \beta^2$$

Propagation constant β:

$$2\beta^2 = (\epsilon_d + \epsilon_m) k_0^2 - \left(\frac{\epsilon_d^2}{\epsilon_m^2} + 1\right) k_{mx}^2 = (\cdot) k_0^2 - \left(\cdot\right)(\epsilon_m k_0^2 - \beta^2)$$

$$\beta^2 \left(1 - \frac{\epsilon_d^2}{\epsilon_m^2}\right) = \left(\epsilon_d + \cancel{\epsilon_m} - \frac{\epsilon_d^2}{\epsilon_m} - \cancel{\epsilon_m}\right) k_0^2 = \epsilon_d \left(1 - \frac{\epsilon_d}{\epsilon_m}\right) k_0^2$$

$$\beta^2 = \epsilon_d \frac{\cancel{1 - \frac{\epsilon_d}{\epsilon_m}}}{\cancel{(1 - \frac{\epsilon_d}{\epsilon_m})}(1 + \frac{\epsilon_d}{\epsilon_m})} k_0^2 \qquad \Rightarrow \beta^2 = \frac{\epsilon_d \, \epsilon_m}{\epsilon_d + \epsilon_m} k_0^2$$

Surface Plasmon Polariton Condition

So far: Brewster angle for lossless media with $\epsilon_r = \text{const}$ re-derived.

No confinement to boundary for $\epsilon_d + \epsilon_m > 0$:

$$k_{dx}^2 = \frac{\epsilon_d^2}{\epsilon_d + \epsilon_m} k_0^2, \qquad k_{mx}^2 = \frac{\epsilon_m^2}{\epsilon_d + \epsilon_m} k_0^2, \qquad \beta^2 = \frac{\epsilon_d \, \epsilon_m}{\epsilon_d + \epsilon_m} k_0^2$$

So far: Correctly assuming $\epsilon_r = \text{const}_\omega$. Now cheating, $\epsilon_r \to \epsilon_r(\omega)$!

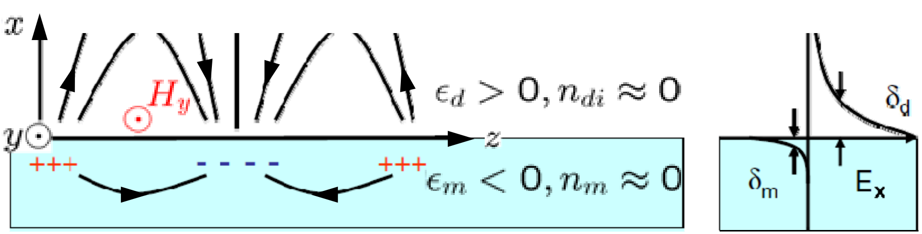
Lossless metal:

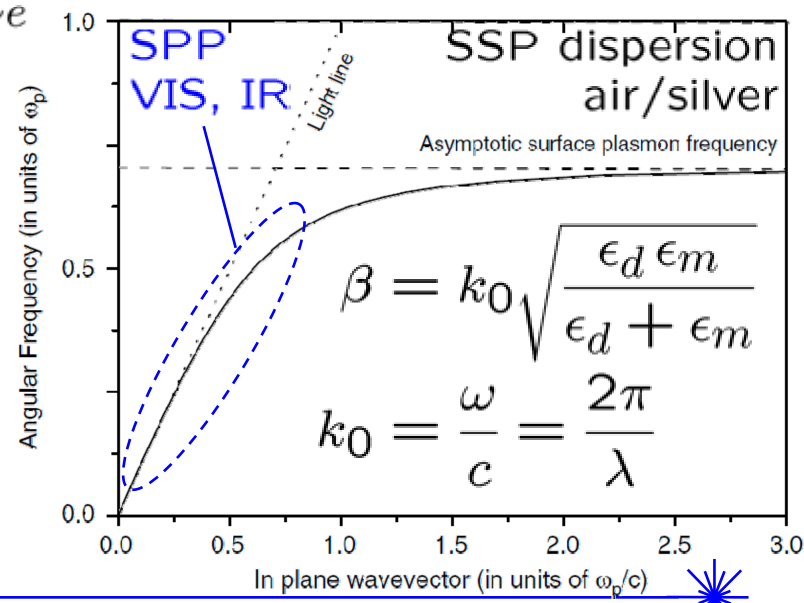
$$\epsilon_m(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \qquad \omega_p^2 = \frac{N f_e \, e^2}{\epsilon_0 m_e} \quad \text{(plasma frequency)}$$

Confinement and propagation (SPP)

for $\epsilon_d + \epsilon_m(\omega) < 0$ and $\epsilon_d > 0$:

$$k_{dx}^2, k_{mx}^2 < 0, \qquad \beta^2 > 0$$





Dielectric-Metal Interface — Dispersion Relation

Propagation constants in x and z-direction ($\epsilon_m k_{dx} = \epsilon_d k_{mx}$):

$$k_{dx}^2 = \frac{\epsilon_d^2}{\epsilon_d + \epsilon_m} k_0^2, \qquad k_{mx}^2 = \frac{\epsilon_m^2}{\epsilon_d + \epsilon_m} k_0^2, \qquad \beta^2 = \frac{\epsilon_d\,\epsilon_m}{\epsilon_d + \epsilon_m} k_0^2$$

Lossless metal, dielectric $\epsilon_d > 0$:

$$\epsilon_m(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \qquad \omega_p^2 = \frac{N f_e\, e^2}{\epsilon_0 m_e} \quad \text{(plasma frequency)}$$

Surface plasmon ($\epsilon_d + \epsilon_m = 0$):

$$\omega_{\mathrm{sp}} = \frac{\omega_p}{\sqrt{\epsilon_d + 1}} \quad \text{for} \quad \beta(\omega_{\mathrm{sp}}) \to \infty$$

$$k_{mx}(\omega_{\mathrm{sp}}) \to \infty$$

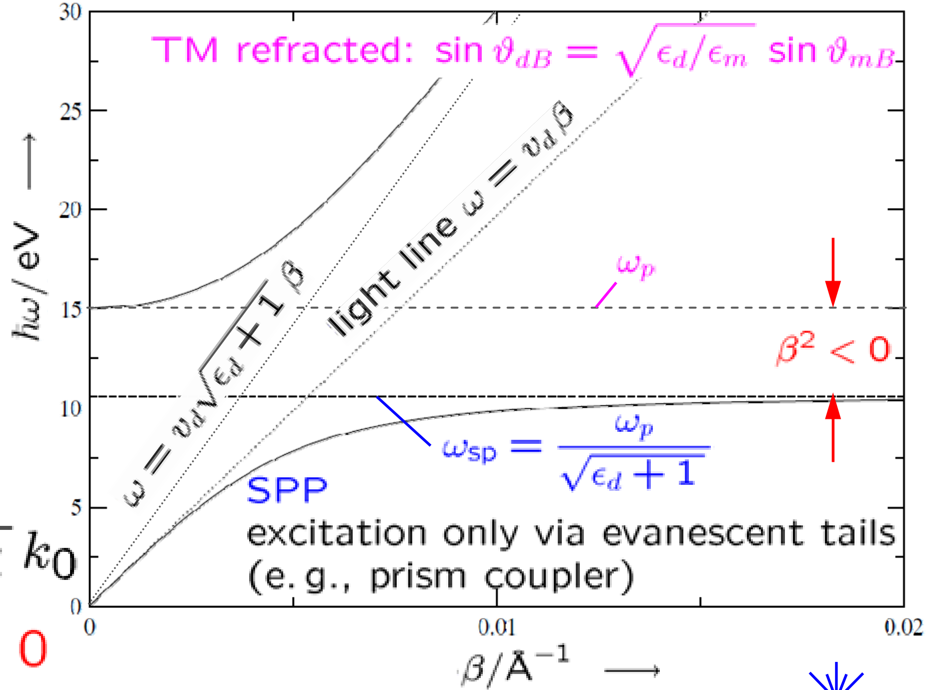
$$v_{\mathrm{sp}\,g} = \mathrm{d}\omega/\mathrm{d}\beta = 0 \quad k_{dx}(\omega_{\mathrm{sp}}) \to \infty$$

Plasma frequency ($\epsilon_m = 0$):

$$\omega = \omega_p \quad \text{for} \quad \beta(\omega_p) = 0$$

$$k_{mx}(\omega_p) = 0$$

$$k_{dx}(\omega_p) = \sqrt{\epsilon_d}\, k_0$$



$$\epsilon_m = n_m^2 - n_{mi}^2, \quad \epsilon_{mi} = 2 n_m n_{mi} = 0$$

SPP Penetration Depths

Propagation constants in x and z-direction ($\epsilon_m k_{dx} = \epsilon_d k_{mx}$):

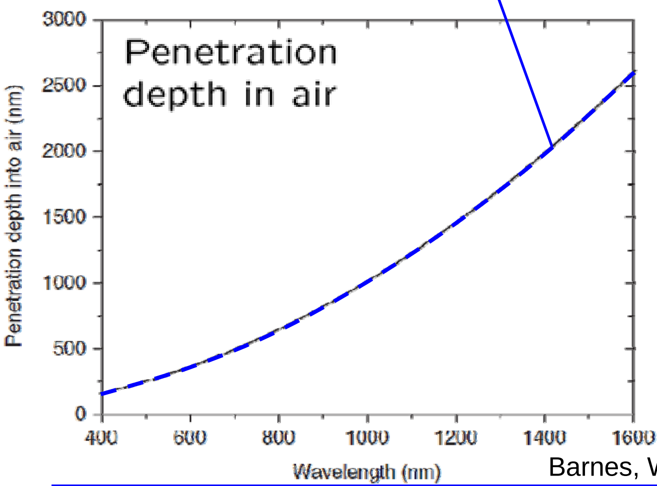
$$k_{dx}^2 = \frac{\epsilon_d^2}{\epsilon_d + \epsilon_m} k_0^2, \qquad k_{mx}^2 = \frac{\epsilon_m^2}{\epsilon_d + \epsilon_m} k_0^2, \qquad \beta^2 = \frac{\epsilon_d \, \epsilon_m}{\epsilon_d + \epsilon_m} k_0^2$$

Lossless metal, dielectric $\epsilon_d > 0$:

$$\epsilon_m(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \qquad \omega_p^2 = \frac{N f_e \, e^2}{\epsilon_0 m_e} \quad \text{(plasma frequency)}$$

Penetrations depths for frequencies $\omega \ll \omega_{\text{sp}}$, $\epsilon_d + \epsilon_m \approx \epsilon_m \approx -\frac{\omega_p^2}{\omega^2}$:

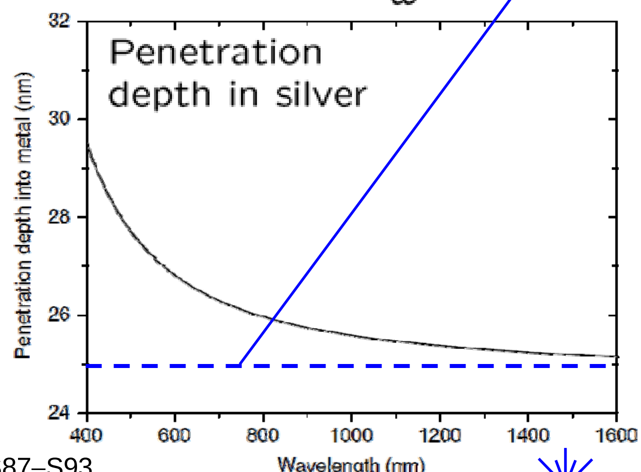
$$\delta_d = \frac{1}{|k_{dx}|} \approx \frac{\frac{\omega_p}{\omega}}{\epsilon_d} \frac{c}{\omega} = \frac{c\,\omega_p}{\epsilon_d} \frac{1}{\omega^2} = \frac{\omega_p}{4\pi^2 \epsilon_d c} \lambda^2 \qquad \delta_m = \frac{1}{|k_{mx}|} \approx \frac{1}{\frac{\omega_p}{\omega}} \frac{c}{\omega} = \frac{c}{\omega_p}$$



Silver:

$$\epsilon_{\text{Ag}} = 1 - \frac{\omega_p^2}{\omega^2 - \mathrm{j}\,\Gamma\omega},$$

$$\omega_p = 2\pi \times 1.91\,\text{PHz}$$
$$\hat{=}\ 7.9\,\text{eV}$$
$$\lambda_p = 157\,\text{nm}$$
$$\Gamma = 1.45 \times 10^{13}\,\text{s}^{-1}$$
$$\hat{=}\ 60\,\text{meV}$$



Barnes, W. L.: J. Opt. A: Pure Appl. Opt. 8 (2006) S87–S93

SPP Wavelength and Propagation Length

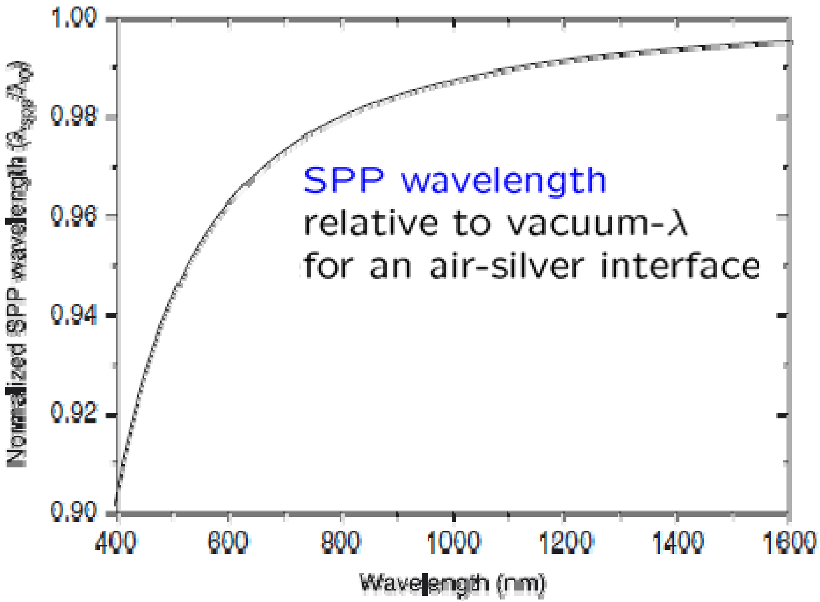


Figure 4. Showing how the normalized surface plasmon–polariton wavelength (i.e. SPP wavelength relative to free space wavelength λ_{SPP}/λ_0) varies with free space wavelength in the visible and near-infrared. The data were computed using the Drude approximation for the relative permittivity of the metal (with ω_p and Γ for silver taken to be $\omega_p = 1.2 \times 10^{16}$ rad s^{-1} ($\equiv$7.9 eV) and $\Gamma = 1.45 \times 10^{13}$ s^{-1} ($\equiv$0.06 eV) respectively). The relative permittivity of the dielectric was taken to be equal to 1. Note that in this spectral range the SPP wavelength is only slightly less than the free space wavelength.

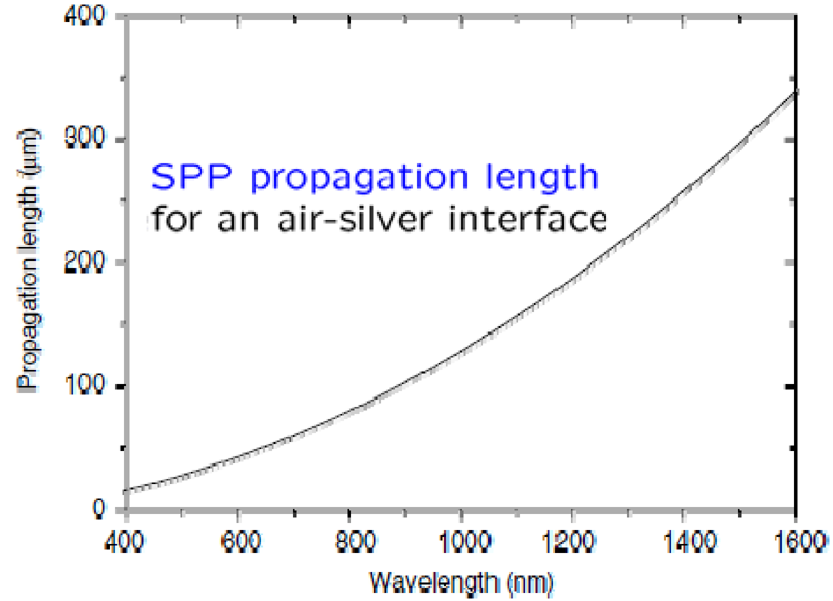
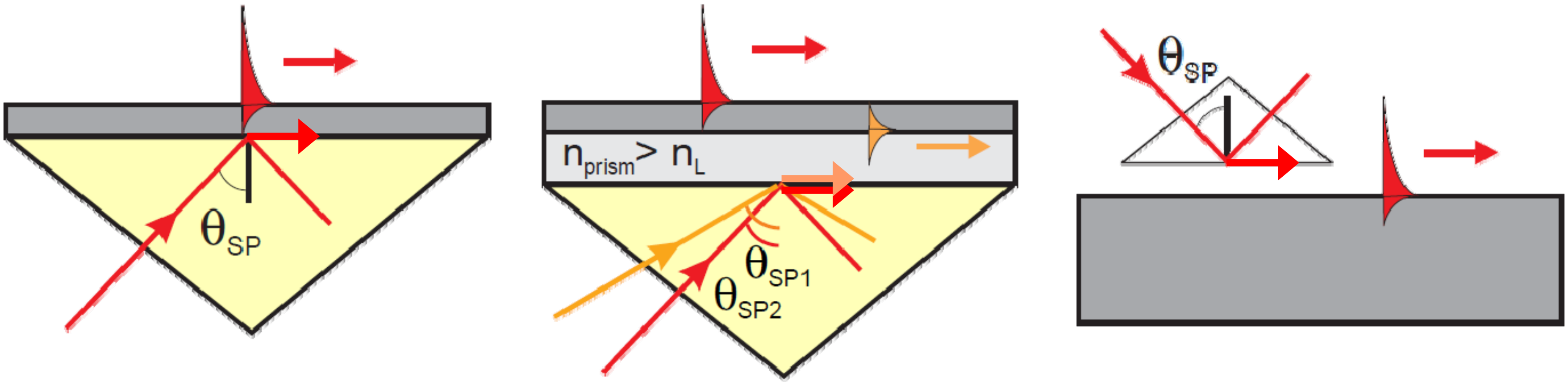


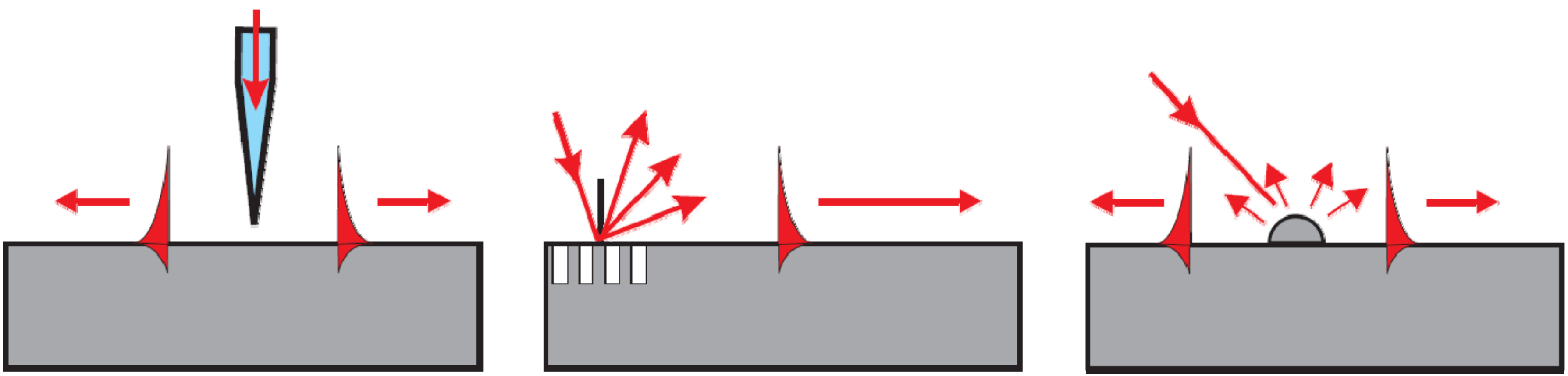
Figure 5. Showing the propagation length of the surface plasmon–polariton. The date were computed using the Drude approximation for the relative permittivity for the metal (with ω_p and Γ for silver taken to be $\omega_p = 1.2 \times 10^{16}$ rad s^{-1} ($\equiv$7.9 eV) and $\Gamma = 1.45 \times 10^{13}$ s^{-1} ($\equiv$0.06 eV) respectively). The relative permittivity of the dielectric was taken to be equal to 1. Note that in calculating these data we have assumed that there is no radiative damping, specifically that there is no mechanism by which SPPs can be converted to light, e.g. by the presence of a prism or grating coupler.

Barnes, W. L.: Surface plasmon-polariton length scales: a route to sub-wavelength optics. J. Opt. A: Pure Appl. Opt. 8 (2006) S87–S93

Surface Plasmon Polariton Excitation

Prism couplers: Frustated total internal reflection

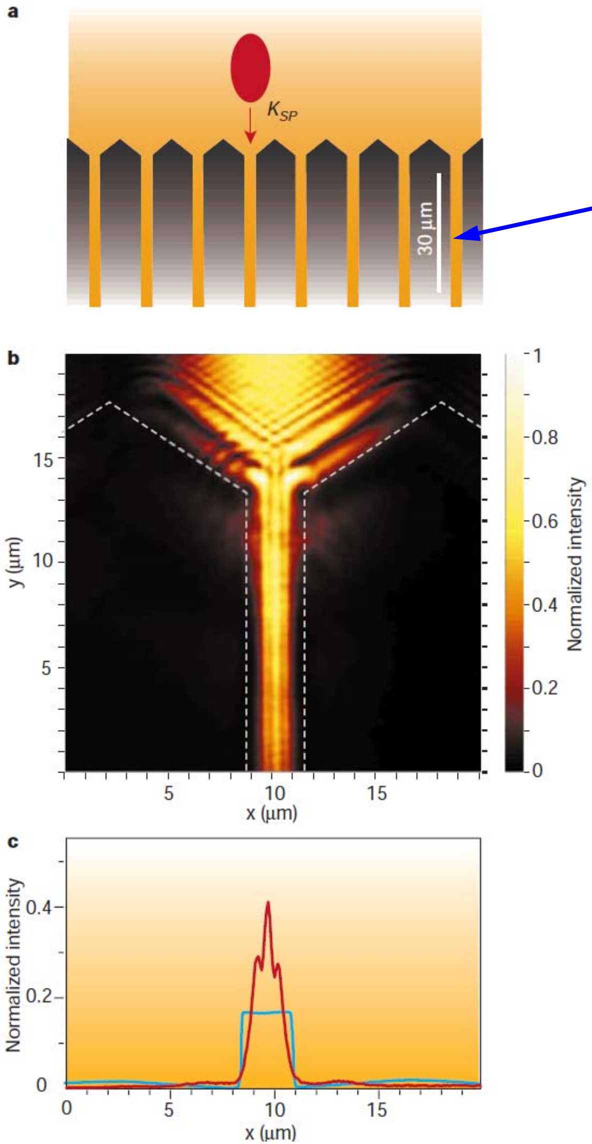




Diffraction couplers: SNOM, grating, scattering at rough surface

Zayats, A. V.; Smolyaninov, I. I.; Maradudin, A. A.: Nano-optics of surface plasmon polaritons. Physics Reports 408 (2005) 131–314

Surface Plasmon Polariton Seen by SNOM



An SP waveguide:
a, A scanning electron micrograph image of a 40 nm thick, 2.5 μm wide gold stripe lying on a glass substrate.

b, The optical functionality of the stripe visualized by photon scanning tunnelling microscopy (PSTM) or scanning nearfield optical microscopy (SNOM); an extended SP, launched on the larger area by a spot (indicated by the red ellipse) generated by total internal reflection illumination ($\lambda = 800$ nm) through the substrate, is used to excite one of the stripe's SP eigen modes featuring three maxima. The PSTM image demonstrates that SPs are bound to the metal.

c, A cross-section across the stripe shows that this mode is much better confined to the guiding material (indicated by the AFM topology — pale blue line) sustaining the mode than would be the case in dielectric-based waveguides.
Not only the height of the guide but also the square root of the waveguide cross-section features a subwavelength size, underlining the fact that the SP mode is essentially bound to the metal surface rather than being a standing wave confined inside the metal volume.

Barnes, W. L.; Dereux, A.; Ebbesen, T. W.: Surface plasmon subwavelength optics. Nature 424 (2003) 824–830

Outline

- Plasma density oscillation
  - Electron density perturbation
  - Plasma frequency and dispersion diagram
- Modeling the dielectric constant
  - Bound and free charge carriers
  - Free carriers and dielectric constant
- Dielectric-dielectric interface
  - Boundary conditions
  - Fresnel's formulae. Brewster's angle
- Dielectric-metal interface
  - Boundary conditions. Surface plasmon polariton (SPP)
  - Dispersion diagram. Characteristic lengths
  - SSP excitation. SNOM visualization
- Summary

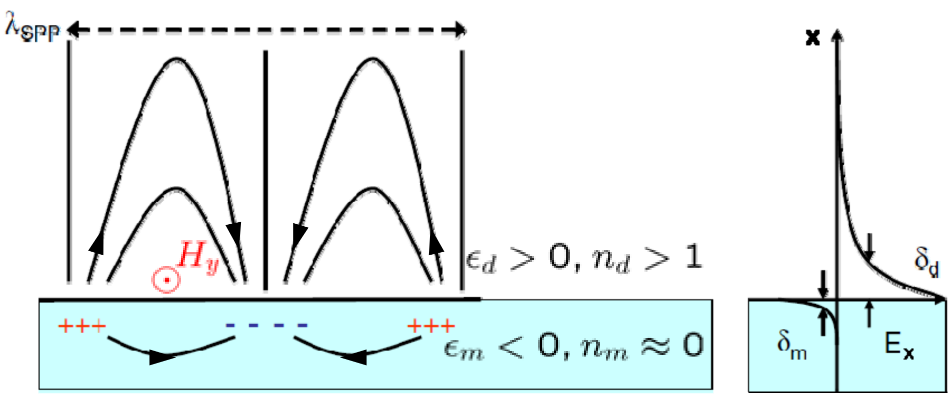
Surface Plasmon Polariton — Overview

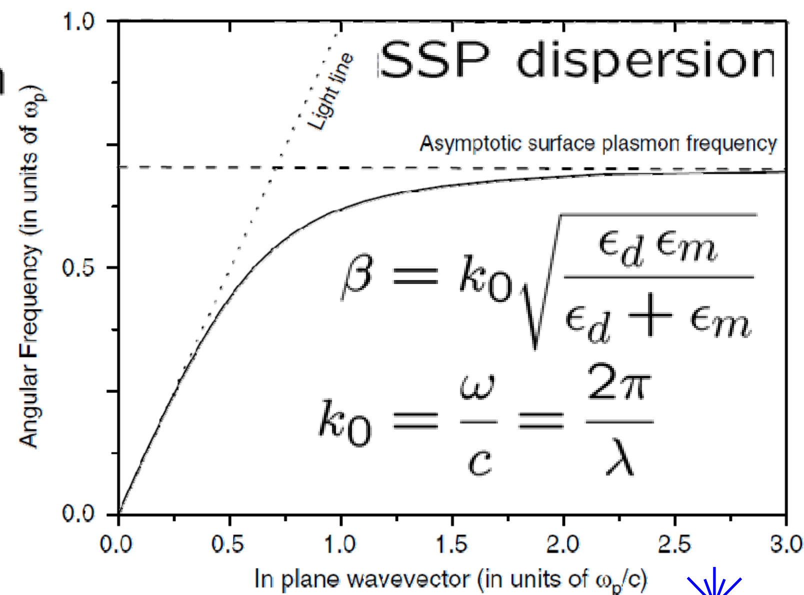
Surface plasmon polariton (SPP) [1]–[10]

- SPP history is older than 100 years (Drude 1889–1904 [1]–[3]).
- Plasmonic waves are collective electronic excitations generated by an electromagnetic field exciting a metal/dielectric interface.
- Fields confined to and propagating along interface → SPP

SPP length scales (including losses; data and figures from [8]):

- Spat. period $\lambda_{\text{SSP}} < \lambda \approx 1\,\mu\text{m}$
- Penetr. depth $\delta_{m;d} \approx 30; 300\,\text{nm}$
- Prop. length $\delta_{\text{SPP}} \approx 30 \ldots 300\,\mu\text{m}$
  long reach $\delta_{\text{LRSPP}} \lesssim 3\,000\,\mu\text{m}$





Surface Plasmon Polariton — Boundary Conditions

SPP length scales (with losses):

- Spat. period $\lambda_{\mathrm{SSP}} < \lambda \approx 1\,\mu\mathrm{m}$
- Penetr. depth $\delta_{m;d} \approx 30; 300\,\mathrm{nm}$
- Prop. length $\delta_{\mathrm{SPP}} \approx 30 \ldots 300\,\mu\mathrm{m}$

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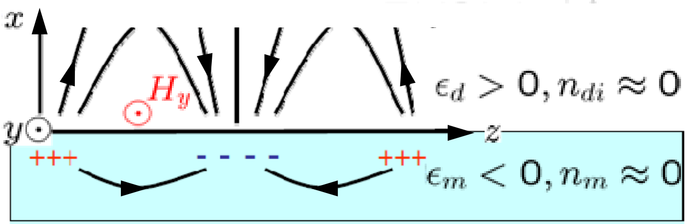
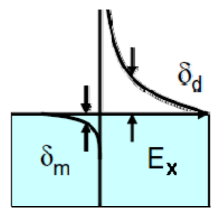
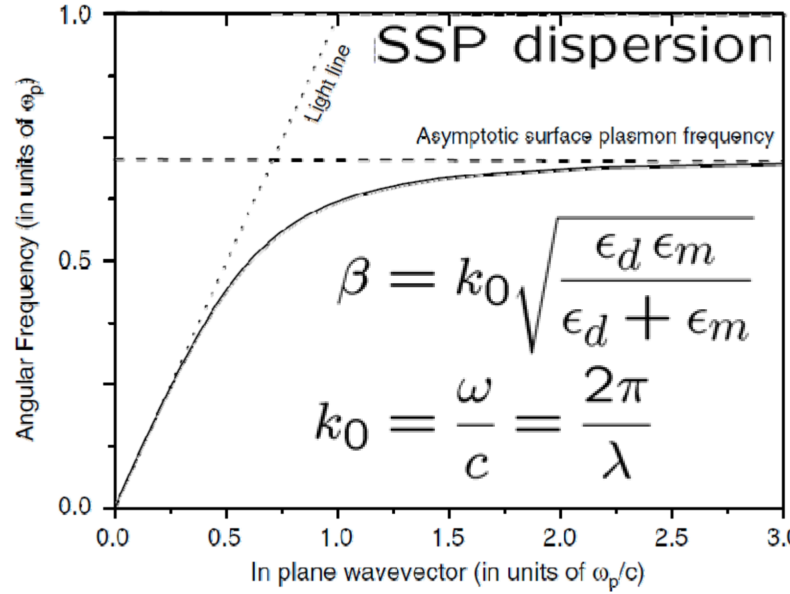






Boundary conditions (SPP involves *charges* at metal surface):

- Normal electric field (E_x) *changes sign* at interface.
- Normal displacement $(D_x = \epsilon_0 \epsilon_r E_x)$ continuous $\rightarrow \epsilon_m < 0$
- Example: $\epsilon_{\mathrm{Au}} \approx -29 - \mathrm{j}\,2.1$ at $\lambda = 830\,\mathrm{nm}$

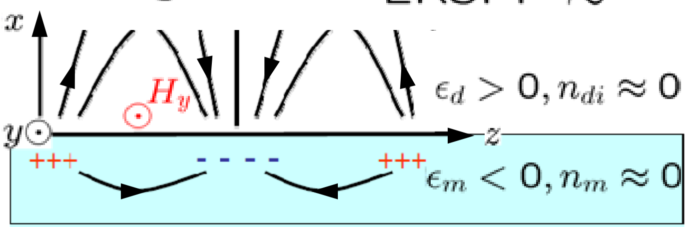
  $\bar{\epsilon}_r = \epsilon_r - \mathrm{j}\,\epsilon_{ri} = (n - \mathrm{j}\,n_i)^2 = n^2 - n_i^2 - \mathrm{j}\,2nn_i$, "no" losses:

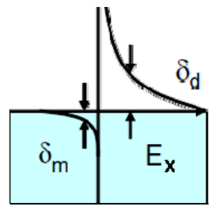
  real $\bar{\epsilon}_r = n^2 - n_i^2$, $\epsilon_{ri} = 2nn_i = 0 \rightarrow \bar{\epsilon}_r = \begin{cases} n^2 & \text{for } n_i = 0 \\ -n_i^2 & \text{for } n = 0 \end{cases}$

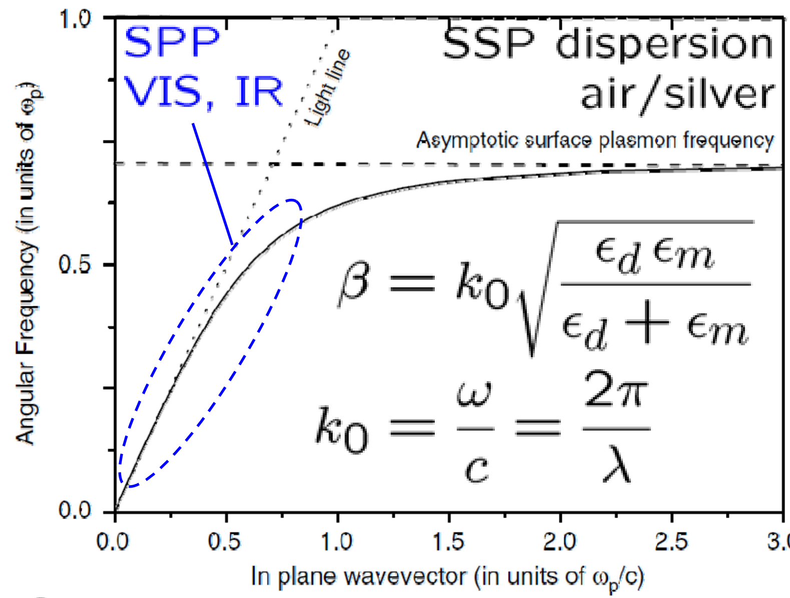
Surface Plasmon Polariton — Propagation Results

SPP length scales (with losses):

- Spat. period $\lambda_{\mathrm{SSP}} < \lambda \approx 1\,\mu\mathrm{m}$
- Penetr. depth $\delta_{m;d} \approx 30; 300\,\mathrm{nm}$
- Prop. length $\delta_{\mathrm{SPP}} \approx 30 \ldots 300\,\mu\mathrm{m}$
  long reach $\delta_{\mathrm{LRSPP}} \lesssim 3\,000\,\mu\mathrm{m}$







$\bar{\epsilon}_r = \epsilon_r - \mathrm{j}\,\epsilon_{ri} = (n - \mathrm{j}\,n_i)^2 = n^2 - n_i^2 - \mathrm{j}\,2nn_i$, "no" losses:

real $\bar{\epsilon}_r = n^2 - n_i^2$, $\epsilon_{ri} = 2nn_i = 0 \rightarrow \bar{\epsilon}_r = \begin{cases} n^2 & \text{for } n_i = 0 \\ -n_i^2 & \text{for } n = 0 \end{cases}$

Propag. const. $\beta > k_0\sqrt{\epsilon_d}$ shows that SPP-mode is non-radiative.

No coupling from outside! Coupling with, e. g., a prism coupler.

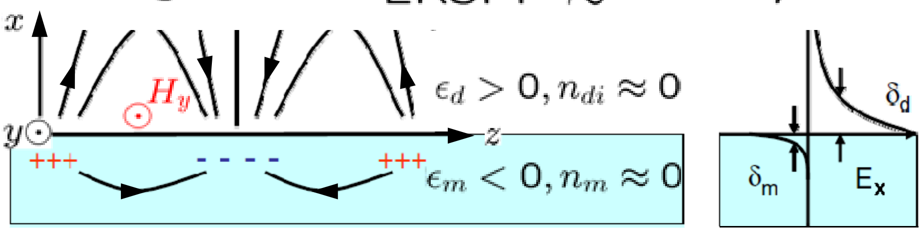
Dielectr. const. in metal $\epsilon_m = 1 - \frac{\omega_p^2}{\omega^2}$ more negative with $\omega \Downarrow$. Limiting case $\epsilon_d + \epsilon_m = 0$, then $\beta \rightarrow \infty$, $\lambda_{\mathrm{SSP}} \rightarrow 0$ for $\omega \rightarrow \omega_p/\sqrt{\epsilon_d + 1}$.

Surface Plasmon Polariton — Summary

SPP length scales (with losses):

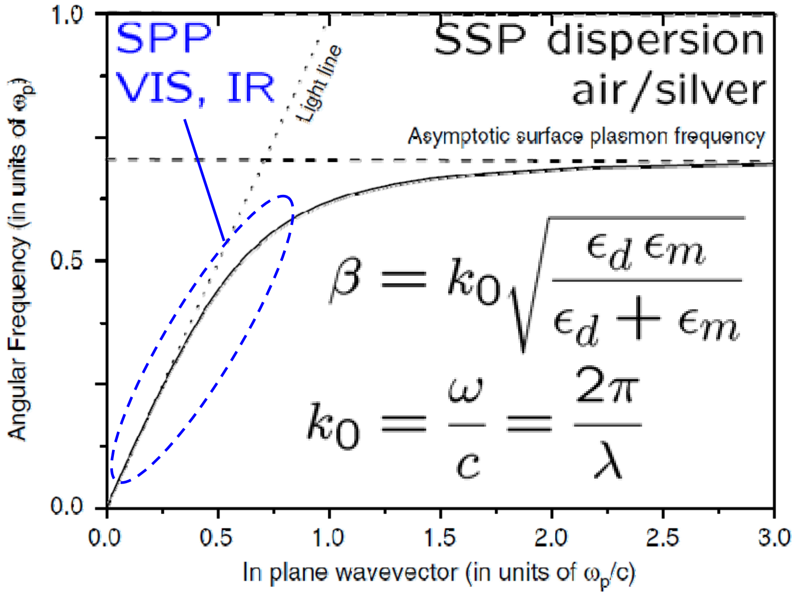
- Spat. period $\lambda_{\mathrm{SSP}} < \lambda \approx 1\,\mu\mathrm{m}$
- Penetr. depth $\delta_{m;d} \approx 30; 300\,\mathrm{nm}$
- Prop. length $\delta_{\mathrm{SPP}} \approx 30 \ldots 300\,\mu\mathrm{m}$

  long reach $\delta_{\mathrm{LRSPP}} \lesssim 3\,000\,\mu\mathrm{m}$



$\epsilon_d > 0, n_{di} \approx 0$, H_y

$\epsilon_m < 0, n_m \approx 0$



$$\beta = k_0 \sqrt{\frac{\epsilon_d\, \epsilon_m}{\epsilon_d + \epsilon_m}}$$

$$k_0 = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

SPP properties — summary:

- Surface charge → *normal* electric field E_x, consequently also E_z: E-wave, TM wave with H_y. No H-wave ($\widehat{=}$ TE wave)!
- Because sign of E_x changes: Field enhancement at boundary
- $n_m \approx 0$, $n_{mi}^2 = |\epsilon_m|$ → no x-propagation, penetratation depths $\delta_d \approx \lambda / n_d$ and $\delta_m \approx \lambda / n_{mi}$ (= skin depth)
- Momentum mismatch $\hbar\beta \geq \hbar k_d$ → guiding → coupling issue

Further Reading (1/2)

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(Papers nowhere cited with title or full pages, not electronically available: Who read it?)

[2] Drude, P.: The theory of optics. London: Longmans 1902

[3] Drude, P.: Optische Eigenschaften und Elektronentheorie, I. & II. Teil. Ann. Physik 14 (1904) 677–725 & 936–961

[4] Born, M.; Wolf, E.: Principles of optics, 6. Ed. Oxford: Pergamon Press 1980.
Erratum: The 1964 edition contains misprints in Sec. 13.2. Eq. (12)–(14): the subscripts for perpendicular and parallel polarization should be interchanged. *Erratum noted by* Nestell, Jr., J. E.; Christy, R. W.: Derivation of optical constants of metals from thin-film measurements at oblique incidence. Appl. Opt. 11 (1972) 643–651, their Ref. [33]

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[6] Krenn, J. R.; Weeber, J.-C.: Surface plasmon polaritons in metal stripes and wires. Phil. Trans. R. Soc. Lond. A 362 (2004) 739–756

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[10] Dragoman, M.; Dragoman, D.: Plasmonics: Applications to nanoscale terahertz and optical devices. Progr. Quantum Electron. 32 (2008) 1–41

Further Reading (2/2)

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[12] Maier, S. A.: Plasmonics: Fundamentals and applications. New York: Springer 2007

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