COLLOCATION METHOD FOR NONLINEAR OPTICAL PULSE PROPAGATION

T. Kremp[1], W. Freude[1] and A. Sharma[2]

[1] High-Frequency and Quantum Electronics Laboratory,
University of Karlsruhe, Kaiserstr. 12, 76128 Karlsruhe, Germany
Tel: +49 721 608 7173, Fax: +49 721 608 2786, E-mail: T.Kremp@ihq.uni-karlsruhe.de
[2] Physics Department, Indian Institute of Technology, Hauz Khas, New Delhi 110016, India

Available photonic design tools usually employ the split-step Fourier method for simulating pulse propagation in nonlinear optical fibres. Since most of the computing time is spent for these routines, any speed improvement will be most welcome. We report on the collocation method, which offers a significant improvement in accuracy and speed.

Introduction

The split-step Fourier method [1] as an extension of the beam propagation method (BPM) is a widely used numerical tool to simulate pulse propagation in nonlinear dispersive optical fibres by solving the generalised nonlinear Schrödinger equation (GNLSE). This technique handles the nonlinear operators in the time domain, and the linear operators in the frequency domain.

We report on an alternative approach based on the collocation principle [2]. The GNLSE is converted into a matrix differential equation, which can be solved using numerical techniques such as the Runge-Kutta method.

Propagation Equation

The basic nonlinear Schrödinger equation (NLSE)

$$\mathrm{j}\frac{\partial \mathrm{E}}{\partial \zeta} = \frac{\partial^2 \mathrm{E}}{\partial \tau^2} + \left|\mathrm{E}\right|^2 \mathrm{E} \qquad (1)$$

as a special case of the more general GNLSE describes the propagation of the normalised slowly varying envelope $\mathrm{E}(\zeta,\tau)$ of the electric field in a dispersive nonlinear element, e.g., an optical fibre. Here, ζ is the normalised propagation length, and τ is the normalised reduced time in a window frame moving with group velocity.

Collocation Method

The principle of the collocation method is the expression of the unknown function $\mathrm{E}(\zeta,\tau)$ by a linear combination of a set of orthogonal functions $\varphi_n(\tau)$,

$$\mathrm{E}(\zeta,\tau) = \sum_{n=1}^{N} c_n(\zeta)\,\varphi_n(\tau)\,. \qquad (2)$$

The unknown coefficients $c_n(\zeta)$ are determined by satisfying the differential equation (1) exactly at N collocation points τ_n. In this way, the differential equation (1) is converted into a set of total differential equations which can be written in matrix form [2],

$$\mathrm{j}\frac{\partial \mathbf{E}}{\partial \xi} = \mathbf{BA}^{-1}\mathbf{E} + \mathbf{PE}\,, \qquad (3)$$

$$\mathrm{A}_{kl} = \varphi_l(\tau_k), \qquad \mathrm{B}_{kl} = \left.\frac{\mathrm{d}^2\varphi_l}{\mathrm{d}\tau^2}\right|_{\tau=\tau_k}, \qquad \mathrm{P}_{kk} = \left|\mathrm{E}_k\right|^2. \qquad (4)$$

Equation (3) can be solved by the Runge-Kutta method, or directly via symmetrized splitting according to

$$\mathbf{E}(\zeta+\Delta\zeta,\tau) \approx \mathrm{e}^{\mathbf{BA}^{-1}\frac{\Delta\zeta}{2\mathrm{j}}}\,\mathrm{e}^{\mathbf{P}\frac{\Delta\zeta}{\mathrm{j}}}\,\mathrm{e}^{\mathbf{BA}^{-1}\frac{\Delta\zeta}{2\mathrm{j}}}\,\mathbf{E}(\zeta,\tau)\,. \qquad (5)$$

The computational effort for solving Eq. (5) is reduced drastically by the fact that **P** is a diagonal matrix. Further, **B** and **A** are independent of ζ and thus need to be evaluated only once before the first propagation step.

Results

As a numerical example, we propagate the fundamental (sech^2)-soliton which is an analytical solution of Eq. (1) over a distance of $\zeta = 10$, corresponding to about 200 km for a pulse width of 20 ps in a standard single mode fibre. Since only the phase changes with ζ, we define the quantity ε as a measure for the numerical error at the fibre end with the help of the modulus of the overlap (coupling) integral c,

$$\varepsilon = 1 - |\,c\,| = 1 - \frac{\left|\int \mathrm{E}^*(\zeta{=}10,\tau)\,\mathrm{E}(\zeta{=}0,\tau)\,\mathrm{d}\tau\right|}{\sqrt{\int \left|\mathrm{E}(\zeta{=}10,\tau)\right|^2 \mathrm{d}\tau \int \left|\mathrm{E}(\zeta{=}0,\tau)\right|^2 \mathrm{d}\tau}}\,. \qquad (6)$$

Figure 1 shows the minimum error for a given computation time, obtained by varying the discretization in ζ and τ over 1…200 propagation steps, 4…65 collocation points, and 8…1024 FFT points for the split-step Fourier method.

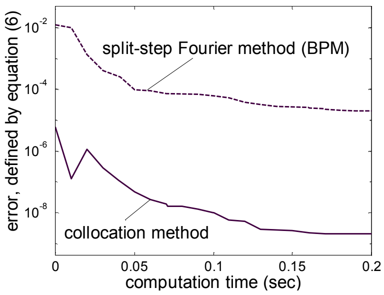

Figure 1: Minimum error for a specified computation time

Compared to the split-step Fourier method, the collocation method *improves the accuracy by about three and a half orders of magnitude* for the same computation time. For the same precision, the collocation method is at least *three times faster than the split-step Fourier method*. Therefore, the collocation method has the potential to replace the split-step Fourier technique as the standard numerical tool for solving nonlinear pulse propagation.

References

/1/ G. P. Agrawal, Nonlinear Fiber Optics. Academic Press San Diego, 2nd Ed. 1995, Chapter 2.4.1, p. 51

/2/ S. Deb, A. Sharma, 'Nonlinear pulse propagation through optical fibers: an efficient method'. Opt. Eng. **32**, 695-699 (1993). Errata: **32**, 2986 (1993).