Finite-Difference Time-Domain and Beam Propagation Methods for Maxwell’s Equations:
Demonstrations with BeamPROP / FullWAVE

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Exercise Overview

• Beam propagation method (BPM)
  1) Modes of a Silicon-on-insulator strip waveguide (WG)

• Finite-difference time-domain (FDTD) method
  2) Stability of the FDTD method
  3) Numerical dispersion of the FDTD method
  4) Band diagram and mode field of a W0.75 photonic crystal (PC)-WG
  5) Coupling of a PC-WG to a strip-WG
  6) Resonance frequency and field distribution of a PC resonator
1) Modes of a Silicon-on-insulator strip WG
Project path & name: 1_BPM_StripWG_SOI

Mode calculation with 3D semivectorial BPM, correlation method. Initial field is a spatial Gaussian pulse off center.
\( \lambda = 1.55 \, \mu m \), discretization \( 0.02 \times 0.02 \times 0.05 \, \mu m^3 \)

Two runs: First for mode spectrum, second for field profiles
All result data is saved in project folder; all displayable result graphs are associated to Winplot and can directly be opened.

Insert output file name (otherwise result not saved)
2) Stability of the FDTD method

Project path & name: 2_FDTD_PlaneWave_Stability

Spatial plane wave, temporal Gaussian pulse propagating through medium, 2D calculation

In middle of medium, a narrow stripe with lower refractive index Δn=0.02 is introduced

\[
2D \text{ stability limit: } S = \frac{\sqrt{2}c\Delta t}{n\Delta x} \leq 1 \Rightarrow c\Delta t \leq \frac{n\Delta x}{\sqrt{2}}
\]

For chosen spatial grid, time step must satisfy limit for lowest refractive index, otherwise risk of exponential increase of wave (instability).

Here: Time step set such that stability is met for “red” material, but not for “blue” stripe

Symbol table:

Variable “Perturbation” switches on (1) or off (0)

Δn for stripe

(Check stability factor “S_red” and “S_blue” for the two different materials)

Run simulations and observe stability and instability
3) Numerical dispersion of the FDTD method

Project path & name: 3_FDTD_PlaneWave_Num_Dispersion

Spatial plane wave, temporal Gaussian pulse propagating through medium, 2D calculation
Temporal pulse shape monitored at three different positions

Coarse spatial discretization chosen, so that $k_z \Delta z / \pi \approx 0.5$ at center frequency of pulse
⇒ By decreasing S (and thus $c \Delta t = S \cdot n \cdot \Delta x / \sqrt{2}$), group and phase velocity deviate more from true value, and group velocity dispersion is increased
⇒ Numerical pulse is broadened and delayed in comparison to real pulse

For different values of S (0.999, 0.1, change in symbol table), run simulations and compare time monitor results concerning pulse shape and delay.

Note: For S=1 defined for 2D with $\Delta x=\Delta z$ (see previous slide), a wave propagating in z-direction experiences a lower $S_z=0.707$. 
4) Band diagram and mode field of a W0.75 PC-WG

Project path & name: 4_FDTD_PC-LDWG_W0_75_Bands

Unit cell of PC-WG with periodic boundary conditions (PBC) along waveguide direction, temporal δ-impulse to excite all frequencies. 2D calculation using effective index.

For a given phase of the PBG $\varphi = k \cdot a$, the temporal field is monitored, peaks in Fourier spectrum give eigenfrequencies.

Calculation of band diagram: Phase $\varphi = k \cdot a$ is swept, and for each value a FDTD simulation is performed and eigenfrequencies calculated. The frequency points plotted vs. phase give the band diagram.

Most interesting band for the W0.75 PC-WG is near normalized frequency 0.3 (graph can be zoomed in).

Field profile at a certain point: Simulation with corresponding phase value, and spatial monitor introduced. In the area of this monitor, a superposition of modes at their respective resonance frequencies exists. A Fourier transform realizes a spectral filter to display the modal field at the desired frequency.

For one band, the mode fields at phases $k \cdot a / \pi = 0.8$ and 0.4 will be calculated using FDTD_PC-LDWG_W0_75_Eigenmode1.ind and ...Eigenmode2.ind. The field results can be opened with Winplot (if an output prefix has been specified).
5) Coupling of a PC-WG to a strip-WG

Project path & name: 5_FDTD_PC-LDWG_W0_75_ExtCoupling

Characterization of W0.75 PC-WG coupled to strip-WG feeders. 2D calculation using effective index.

Mode of strip-WG with temporal pulse of narrow bandwidth around center frequency excited.

Time monitors behind excitation and at output calculate spatial overlap integral with excitation. Fourier transforms of time monitors show transmission and reflection as a function of frequency.

After the simulation has been run, the monitor results can be opened with Winplot (if an output prefix has been specified). Within the pulse frequency range, two passbands and a stop band of the PC-WG can be observed.

The project files FDTD_PC-LDWG_W0_75_ExtCoupling_CW1.ind and …CW2.ind can start simulations to show the propagation of continuous wave signals in the pass band and in the stop band of the PC-WG.
6) Resonance frequency and field distribution of a PC resonator

Project path & name: 6_FDTD_PC-Resonator_H1

H1 PC resonator, 2D calculation using effective index
As with band diagram simulations: Temporal impulse to excite all frequencies within resonator is used, Fourier transform of temporal time monitor shows resonant frequencies (specify prefix).

For the field profile at resonance, again simulation with introduced spatial monitor. In the area of this monitor, a superposition of modes with respective resonance frequencies exists. A Fourier transform realizes a spectral filter to show the field at the desired resonance frequency.

FDTD_PC-Resonator_H1_field.ind