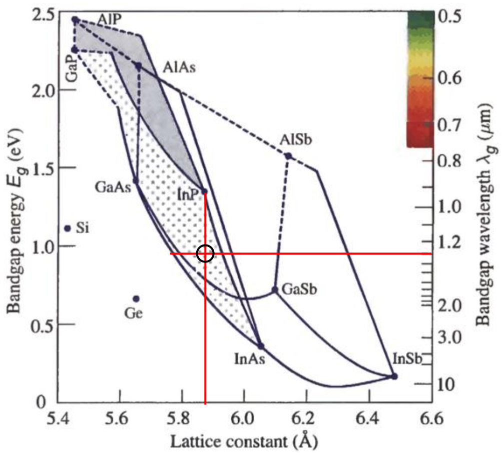
6. Tutorial on Optical Sources and Detectors

June 12th 2012

Problem 1: Lattice-matched compound semiconductors

Imagine you are working for a laser producing company. Your task is to build a laser that is to be grown on an InP substrate and emits slightly above the material's bandgap wavelength of 1300 nm. Specify the chemical composition of the material.



Ternary and quaternary III-V semiconductors. Ternary materials are represented by the line that joins two binary compounds, whereas quaternary materials correspond to the areas that are defined by the binary compound corners.

➔ To be grown on InP the lattice constant of the compound has to be the same as InP. From the diagram we find, that for a bandgap around 1300nm and the lattice constant of InP the desired material system is InGaAsP.

The chemical composition of $(In_{1-x}Ga_x)(As_yP_{1-y})$ for a specific bandgap can be derived from the empirical formula: $W_{G,[eV]} = 1.35 - 0.72y + 0.12y^2$ with $x = y/(2.2091 - 0.06864\,y)$ when lattice matched to InP.

$$W_G = \frac{hc}{\lambda} = 0.954eV = 1.35 - 0.72y + 0.12y^2$$
$$y = 0.613, \quad x = y/(2.2091 - 0.06864\,y) = 0.283$$

Problem 2: Gain measurement in a Fabry-Perot resonator

Figure 1 depicts schematically a Fabry-Perot cavity which consists of two mirrors (power reflection coefficients R_1 and R_2) and an active waveguide of length L = 500 µm. Light propagating in the positive z-direction experiences a phase shift according to $\exp\{-jk_0 n_e z\}$, where k_0 is the free-space wavenumber and n_e denotes the effective refractive index of the waveguide mode. The modal power gain coefficient is given by Γg, where g is the gain coefficient of the material which is used for the active zone and Γ denotes the field confinement factor, i.e. the fraction of the light that actually propagates within the active zone. The modal loss coefficient of the waveguide amounts to α = 25 cm^{-1} which leads to a complex field amplitude according to $b(z) = \exp[0.5(\Gamma g - \alpha)z] \cdot \exp[-\mathrm{j}k_0 n_e z]$ for an optical wave propagating from left to right. The material for the active region is InGaAsP with a peak gain wavelength λ_0 = 1550 nm. Dispersion can be neglected.

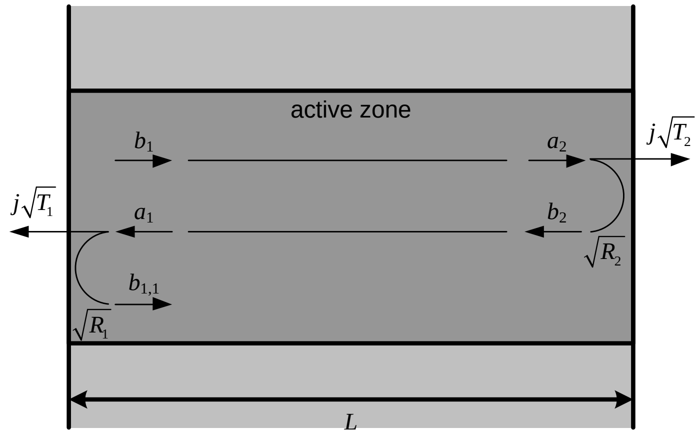


Figure 1: Schematic of a Fabry-Perot laser cavity. The active zone is marked by the area in dark grey. A material of lower refractive index is transversally surrounding the active zone. To the left and right there are mirrors, which could be in the most simple case just cleaved facets.

a) The gain in the waveguide is below threshold, i.e. the losses are larger than the gain. Assume that an electromagnetic field is entering the resonator at the left with a complex amplitude b_1. What is the field $b_{1,1}$ measured at the same position after one roundtrip? What is the field amplitude $b_{1,i}$ after i = ∞ roundtrips? Make use of the geometric series to get a closed expression.

➔ The given field amplitude in the resonator at one facet is b_1, after one roundtrip we get:

$$b_{1,1}(z) = b_1 \cdot \sqrt{R_1 R_2}\, e^{\frac{(\Gamma g - \alpha)2L}{2}} \cdot e^{-\mathrm{j}k_0 n_e 2L}$$

And after $i = \infty$ roundtrips the field is

$$b_{1,\infty} = b_1 \sum_{i=0}^{\infty} (R_1 R_2)^{i/2} e^{\frac{(\Gamma g - \alpha)2L \cdot i}{2}} \cdot e^{-\mathrm{j}k_0 n_e 2L \cdot i}, \quad i \in \mathbb{N}$$

With the geometric series $\sum_{i=0}^{\infty} aq^i = \frac{1}{1-q}$ for $|q| < 1$ we find:

$$b_{1,\infty} = b_1 \sum_{i=0}^{\infty} \left[(R_1 R_2)^{1/2} e^{\frac{(\Gamma g - \alpha)2L}{2}} \cdot e^{-\mathrm{j}k_0 n_e 2L} \right]^i = \frac{b_1}{1 - \sqrt{R_1 R_2}\, e^{\frac{(\Gamma g - \alpha)2L}{2}} \cdot e^{-\mathrm{j}k_0 n_e 2L}}$$

Since we assume that the losses α are larger than the gain Γg, the term in brackets is smaller than 1 and the geometric series converges.

b) Using a method shown by B. W. Hakki, and T. L. Paoli, “Gain spectra in GaAs double-heterostructure injection lasers,” JAP **46**(3), 1299-1306 (1975), it is possible to determine the material gain inside the resonator from the contrast between the minima and maxima of the amplified spontaneous emission (ASE). To understand this, let us assume that we can model the spontaneous emission inside the resonator as a complex field amplitude b_1 launched at the left side of the resonator, which is the amplified over several round-trips in the resonator. Depending on the modal propagation constant $k_0 n_e$ the field components after several roundtrips can sum up constructively or destructively. For a constructive interference the phase factor for one roundtrip must equal $\exp\{-\mathrm{j}k_0 n_e 2L\}=1$, while for destructive interference the phase factor must be $\exp\{-\mathrm{j}k_0 n_e 2L\}=-1$. Note that the power P measured outside the resonator is proportional to the squared magnitude of the internal field amplitude, $P \sim |b_{1,\infty}|^2$.

Show that the net gain in the resonator can be related to the maximum and minimum power levels in the Fabry-Perot spectrum. Using the results from a) and the relation $\frac{\sqrt{P_{max}}}{\sqrt{P_{min}}}$ you should come to the result:

$$\Gamma g-\alpha=\frac{1}{L}\ln\left(\frac{\sqrt{P_{max}}-\sqrt{P_{min}}}{\sqrt{P_{max}}+\sqrt{P_{min}}}\right)-\frac{1}{L}\ln\left(\sqrt{R_1R_2}\right)$$

➔ Using the expression for the fields from a) we get for the maximum and minimum

$$b_{1,\infty,max}=b_{1,\max}\frac{1}{1-\sqrt{R_1R_2}e^{\frac{(\Gamma g-\alpha)2L}{2}}}$$

$$b_{1,\infty,min}=b_{1,\min}\frac{1}{1+\sqrt{R_1R_2}e^{\frac{(\Gamma g-\alpha)2L}{2}}}$$

Since $P \sim |b_{1,\infty}|^2$ we can write for the relation $\frac{\sqrt{P_{max}}}{\sqrt{P_{min}}}=\frac{|b_{1,\infty,max}|}{|b_{1,\infty,min}|}$

$$\frac{\sqrt{P_{max}}}{\sqrt{P_{min}}}=\frac{|b_{1,\infty,max}|}{|b_{1,\infty,min}|}=\frac{|b_{1,max}|}{|b_{1,min}|}\frac{1+\sqrt{R_1R_2}e^{\frac{(\Gamma g-\alpha)2L}{2}}}{1-\sqrt{R_1R_2}e^{\frac{(\Gamma g-\alpha)2L}{2}}}=\frac{1+\sqrt{R_1R_2}e^{(\Gamma g-\alpha)L}}{1-\sqrt{R_1R_2}e^{(\Gamma g-\alpha)L}}$$

$$\sqrt{P_{max}}-\sqrt{P_{max}}\sqrt{R_1R_2}e^{(\Gamma g-\alpha)L}=\sqrt{P_{\min}}+\sqrt{P_{\min}}\sqrt{R_1R_2}e^{(\Gamma g-\alpha)L}$$

$$\sqrt{P_{max}}-\sqrt{P_{\min}}=\left(\sqrt{P_{max}}+\sqrt{P_{\min}}\right)\sqrt{R_1R_2}e^{(\Gamma g-\alpha)L}$$

$$\frac{\sqrt{P_{max}}-\sqrt{P_{\min}}}{\sqrt{P_{max}}+\sqrt{P_{\min}}}=\sqrt{R_1R_2}e^{(\Gamma g-\alpha)L}$$

$$\ln\left(\frac{\sqrt{P_{max}}-\sqrt{P_{\min}}}{\sqrt{P_{max}}+\sqrt{P_{\min}}}\right)=\ln\left(\sqrt{R_1R_2}\right)+(\Gamma g-\alpha)L$$

$$\Gamma g-\alpha=\frac{1}{L}\ln\left(\frac{\sqrt{P_{max}}-\sqrt{P_{\min}}}{\sqrt{P_{max}}+\sqrt{P_{\min}}}\right)-\frac{1}{L}\ln\left(\sqrt{R_1R_2}\right)$$

c) In Figure 2 a spectrum with the normalized power of the ASE is depicted as a function of wavelength. By using the magnitude of the minima and maxima determine the net gain Γg. Use $R_1=R_2=0.3$ and the other values from a).
The spacing between two peaks is $\Delta\lambda = 0.686\text{nm}$, what is the effective group refractive index in this resonator structure?

➔

$$\Gamma g=\frac{1}{L}\ln\left(\frac{\sqrt{P_{max}}-\sqrt{P_{\min}}}{\sqrt{P_{max}}+\sqrt{P_{\min}}}\right)-\frac{1}{L}\ln\left(\sqrt{R_1R_2}\right)+\alpha$$

$$\Gamma g=\frac{1}{500\mu m}\ln\left(\frac{\sqrt{1}-\sqrt{0.4}}{\sqrt{1}+\sqrt{0.4}}\right)-\frac{1}{500\mu m}\ln\left(\sqrt{0.3\cdot 0.3}\right)+25cm^{-1}=19.26cm^{-1}$$

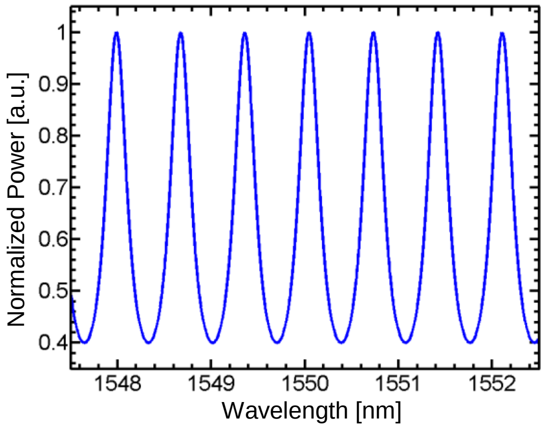


Figure 2: Normalized optical spectrum of a Fabry Perot resonator

➔ The phase factor of the maxima is $\exp\{-\mathrm{j}k_0 n_e 2L\} = 1$ we can derive the condition for adjacent maxima:

$$k_0 n_e 2L = i2\pi, \quad i \in \mathbb{N}$$

$$\frac{2\pi f}{c} n_e 2L = i2\pi$$

$$f n_e = i\frac{c}{2L}$$

When we consider i to be continuous we can write the differential

$$\frac{\mathrm{d}(f n_e)}{\mathrm{d}i} = \frac{c}{2L} = \frac{\mathrm{d}f}{\mathrm{d}i} n_e + f\frac{\mathrm{d}n_e}{\mathrm{d}f}\frac{\mathrm{d}f}{\mathrm{d}i} = \frac{\mathrm{d}f}{\mathrm{d}i}\underbrace{\left(n_e + f\frac{\mathrm{d}n_e}{\mathrm{d}f}\right)}_{n_{eg}} = \frac{c}{2L}$$

Again considering the discrete steps we can write df → Δf and di → 1 which leads to the expression for the free spectral range

$$\Delta f = \frac{c}{2n_{eg}L}, \quad \text{or} \quad \Delta\lambda = \frac{\lambda_0^2}{2n_{eg}L}$$

$$n_{eg} = \frac{\lambda_0^2}{2\Delta\lambda L} = \frac{(1550nm)^2}{2\cdot 0.686nm\cdot 500\mu m} = 3.5$$

Questions and Comments:

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