3. Tutorial on Optical Sources and Detectors

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Problem 1: Wavelength and frequency conversion

Convert the following characteristics as they can be found in the literature (e.g. laser data sheet or ITU-T Recommendation G.694.1). Write down the formulas used for conversion. You can use a calculator however you should remember some order of magnitude estimates!

Characteristic	Converted
Wavelength: 1550 nm	Frequency: 193.5 THz
Frequency: 229 THz	Wavelength: 1310 nm
Spectral line width @ center wavelength of 1550 nm:	$\Delta\lambda = 8 \text{ fm}$
$\Delta f = 1 \text{ MHz}$	
Spectral line width @ center wavelength of 1310 nm:	$\Delta\lambda = 5.7 \text{ fm}$
$\Delta f = 1 \text{ MHz}$	
ITU ¹ frequency spacing (center frequency 193.1 THz):	$\Delta\lambda = 0.8 \text{ nm}$
$\Delta f = 100 \text{ GHz}$	
Resolution bandwidth of an OSA ² (APEX AP2043B,	$\Delta f = 20 \text{ MHz}$
Span: 1526 nm to 1567 nm): $\Delta \lambda = 0.16$ pm	

Problem 2: Erbium doped fiber amplifier

Erbium doped fiber amplifier (EDFA) are widely used devices in optical networks. They consist normally of a single mode fiber which is doped with Erbium, a rare earth element. The fiber is optically pumped with laser diodes at 980 nm or 1480 nm and the resulting gain is used for signal amplification.

a) Typical characteristic data of an EDFA are a spectral width of $\Delta \lambda = 50$ nm at a center wavelength $\lambda_0 = 1.55 \ \mu m$ and a spontaneous lifetime of $\tau_{sp} = 10 \ ms$.

Is the Erbium doped fiber a homogeneously or inhomogeneously broadened gain material? Why?

→ The influence of the variation of the local field near the erbium ions leads to an inhomogeneous broadening. This fact can also be seen from the numbers. In the case of homogeneous broadening the relation must hold:

$$\Delta f = \frac{1}{2\pi \tau_{sp}}$$
, with the given values: $\Delta f = \frac{1}{2\pi 10ms} \approx 16Hz$

¹ ITU: International Telecommunication Union

² OSA: Optical Spectrum Analyzer

But from measurements the spectral width is found to be 50 nm (6.24 THz) which is much larger than the value calculated before. The erbium doped fiber must be therefore inhomogeneously broadened.

b) The refractive index of the fiber is n = 1.46. With the given values from a) determine the emission cross section $\sigma(f)$ of the Er^{3+} atom. For simplicity assume that the line shape function has a rectangular shape of area one and spectral width $\Delta \lambda$ as depicted in Fig. 1.

$$\Rightarrow \qquad \gamma(f) = \frac{\sigma(f)}{\int \sigma(f) df}, \quad \int \sigma(f) df = \frac{\lambda_0^2}{8\pi n^3 \tau_{sp}} = \frac{\sigma(f)}{\gamma(f)}$$

The lineshape function is rectangular in the region of interest $\gamma(f)=1/\Delta f$ and from a) we got $\Delta f = 6.24$ THz

$$\sigma(f) = \frac{(1550nm)^2}{8\pi 1.46^3 \cdot 10ms \cdot 6.24THz} = 4.92 \cdot 10^{-25} m^2 = 4.92 \cdot 10^{-21} cm^2$$

- c) Take the value for the emission cross section and calculate the population difference N_2 - N_1 that is required to achieve 30 dB gain in an EDFA with 30 m of Erbium doped fiber.
 - → A gain of 30dB corresponds to an factor of 1000, for the gain coefficient g in the fiber we can write

$$G = e^{g \cdot L}$$
, $g = \ln(G) / L = \ln(1000) / 30m = 0.23m^{-1}$

$$g = (N_2 - N_1)n\sigma(f), \quad (N_2 - N_1) = \frac{g}{n\sigma(f)} = \frac{0.23m^{-1}}{1.46 \cdot 4.92 \cdot 10^{-25}m^2} = 3.2 \cdot 10^{17} \, cm^{-3}$$

- d) Why are EDFA so popular in wavelength division multiplexing (WDM) systems?
 - → EDFA have little cross gain modulation

Since the fiber can be made quite long they have a large amplification They have a wide gain bandwidth which covers the C-Band, which is widely used in telecommunication.



Fig. 1: Simplified line shape function

Problem 3: Transatlantic fiber link

Without optical amplification it is not possible to optically transmit data from London to New York, i.e. 7000 km of fiber length with an attenuation of 0.2 dB/km. Therefore Erbium doped fiber amplifiers (EDFA), which can amplify an optical input signal of -15 dBm to +1 dBm, are employed for a transatlantic fiber link. Assume that the optical transmitter at London is sending with an optical power of +1 dBm and that the receiver at New York is able to detect optical signals down to -35 dBm.

- a) How many EDFA are necessary for an optical transatlantic fiber link?
 - → One EDFA amplifies from -15 dBm to +1 dBm → 16dB gain. The distance between two EDFA calculates to $d = \frac{16dB}{0.2dB/km} = 80km$. In the last link the losses can be 36 dB, therefore the span can be $d_{last} = \frac{36dB}{0.2dB/km} = 180km$

Minimum number of amplifier:

$$N \cdot 80km + 180km \ge 7000km, \quad N \ge \frac{7000km - 180km}{80km} = 85.25$$

86 EDFA are needed for the transatlantic link

b) Each EDFA needs electrical power supply. Estimate the power consumption of a single EDFA for 8 transmitted channels, if the power efficiency of the EDFA is $\eta = 0.01$ and the power consumption is proportional to the number of transmitted channels. How much is the power consumption for the whole transatlantic fiber link (assume that electrical line losses can be neglected)?

$$\eta = \frac{P_{out,opt}}{P_{in,el}} = 0.01, \quad P_{in,el} = \frac{P_{out,opt}}{\eta} \cdot N_{channels} \cdot N_{EDFA}$$
$$P_{in,el} = \frac{1dBm}{\eta} \cdot 8 \cdot 1 = 1.01W$$
$$P_{in,el} = \frac{1dBm}{\eta} \cdot 8 \cdot 86 = 86.68W$$

Questions and Comments:

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