## 2. Tutorial on Optical Sources and Detectors

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## Problem 1: Density of modes in an optical resonator

Consider a three-dimensional optical resonator constructed of three pairs of parallel mirrors that form the walls of a box with edge lengths $L_{x}, L_{y}$, and $L_{z}$. Eigenmodes of the resonator have to fulfill the boundary conditions at the sidewalls and are represented by standing-wave solutions with discrete components $k_{x}, k_{y}$, and $k_{z}$ of the wavevector. In the following, we will calculate the density of these modes per unit of volume and frequency. The derivation can be performed in analogy to the density of states of electrons in a semiconductor.
a) Find the condition for $k_{x}, k_{y}$, and $k_{z}$ that need to be fulfilled in order to obtain standing waves within the three-dimensional resonator. For simplicity, assume that the resonator is filled with air of unity refractive index.
b) Each pair of modes (considering two polarizations) is represented by a positive triple of $k_{x}, k_{y}$, and $k_{z}$, all of which are fulfilling the boundary condition. Calculate the volume $V_{k}$ that a single optical mode occupies in $k$-space?
c) Calculate the number of optical modes $M(f)$ within the frequency interval 0 and $f$. In $k$-space these modes lie within the positive octant of a sphere, the radius of which is related to $f$.
d) Calculate the density of modes, i.e. the number of optical modes per volume and per frequency interval:

$$
\rho(f)=\frac{1}{V} \frac{d M(f)}{d f}
$$

e) In Section 2.1.2 of the lecture notes the following quantity is denoted as the "average number of photons per mode":

$$
\bar{N}_{p}=\frac{\lambda_{0}^{3}}{8 \pi h n^{3}} u\left(f_{0}\right) .
$$

Show that this formula can be derived by dividing the number of photons $N_{p}$ in a resonator by the number of modes in the resonator. Is the average number of photons per mode dependent on the parameters of the model resonator?

## Problem 2: Carrier Concentration in Semiconductors

The densities $n$ of conduction band (CB) electrons and the density $p$ of valence band (VB) holes of an undoped semiconductor can be calculated with the help of the density of states $\rho_{C}(W)$ of the CB and $\rho_{V}(W)$ of the VB , and the Fermi-Dirac distributions $f(W)$ and [1-f(W)] for electrons and holes, respectively:

$$
\begin{aligned}
& n=\int_{W_{C}}^{\infty} \rho_{C}(W) f(W) \mathrm{d} W \\
& p=\int_{-\infty}^{W_{V}} \rho_{V}(W)[1-f(W)] \mathrm{d} W
\end{aligned}
$$

If the energetic distance of the Fermi level from the band edges is $\left|W_{C, V}-W_{F}\right|>3 k T$, then the Fermi functions can be simplified by the Boltzmann approximation

$$
\begin{array}{r}
f(W)=\exp \left\{-\frac{W-W_{F}}{k T}\right\}, \quad \text { for }\left(W-W_{F}\right)>3 k T, \\
f(W)=1-\exp \left\{-\frac{W_{F}-W}{k T}\right\}, \quad \text { for }\left(W-W_{F}\right)<-3 k T,
\end{array}
$$

which shall be used for the calculations below. In practice, however, this condition is usually not fulfilled, e.g. for a highly doped laser diode, the Fermi level is very close to or even inside the band. Thus the following considerations show the principle only.
a) Show that the maximum of the electron distribution $\rho_{C}(W) f(W)$ is found at the energy $k T / 2$ above the band edge $W_{C}$ of the CB.
b) Solve the above given integrals for $n$ and $p$ by using the Boltzmann approximation and write the carrier concentration as

$$
\begin{aligned}
& n=N_{C} \exp \left(-\frac{W_{C}-W_{\mathrm{F}}}{k T}\right) \\
& p=N_{V} \exp \left(-\frac{W_{\mathrm{F}}-W_{V}}{k T}\right),
\end{aligned}
$$

where $N_{C}$ and $N_{V}$ are the effective density of states of the CB and VB , respectively.
c) Take the results from b) and determine the energetic distance of the Fermi level relative to the band edges.
d) The bandgap energy of GaAs is $W_{G}=1.424 \mathrm{eV}$, the effective masses are $m_{n}=0.067 m_{0}$ and $m_{p}=0.48 m_{0}$, where $m_{0}$ is the electron rest mass. Determine the intrinsic carrier concentrations $n$ and $p$ of GaAs at $\mathrm{T}=300 \mathrm{~K}$ by using the results of b ).

## Problem 3: Fiber optic communication system

Consider a fiber optic communication system that transmits data with a data rate of $40 \mathrm{Gbit} / \mathrm{s}$ over a distance of 100 km . The data is modulated onto a carrier at 1550 nm and the fiber has an average attenuation (considering splices etc.) of $0.3 \mathrm{~dB} / \mathrm{km}$. The transmitter couples an average power of 2 mW into this fiber.
Assume that the ' 0 ' bits don't carry any power whereas the ' 1 ' bits have a constant power throughout the whole bit slot (non-return-to-zero (NRZ) modulation). Further suppose that the probability for transmitting a ' 1 ' is equal to the probability of transmitting a ' 0 '.

How many photons arrive at the receiver during a single '1' bit?

## Questions and Comments:

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